PHYS 5130, Mid-Term 1

1. A three-level laser with the energy levels given in Fig. 1, has an emission bandwidth of 2.14 nm and total state density of 5.46×10^{21} cm^{-3}. If the 1 mm-long gain medium has an absorbance maximum $(A = \alpha L)$ of 0.5, then what is the state density rate $\left(\text{i.e., } \frac{N}{Vt}\right)$ between $|2\rangle$ and $|1\rangle$ (in units of $\frac{1}{\text{cm}^3-\text{s}}$)?

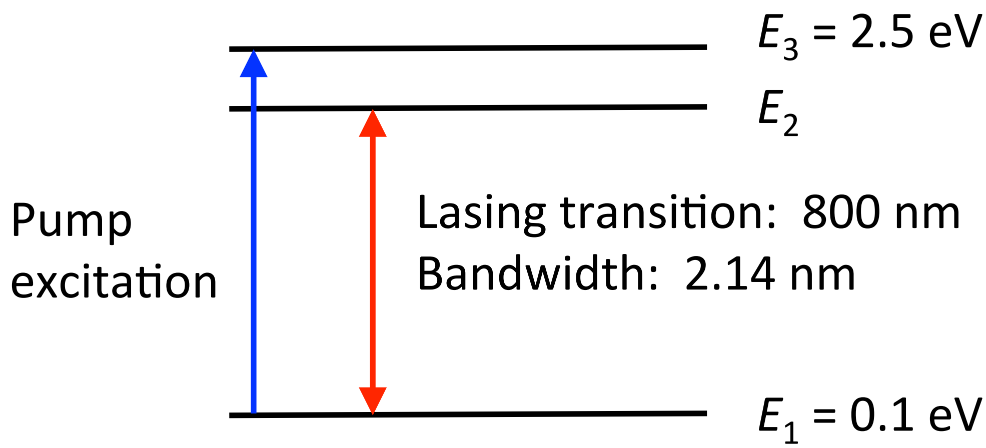

FIG. 1. Energy level schematic of the three-level laser for Problem 1. The lasing transition $|2\rangle \rightarrow |1\rangle$ has a wavelength of 800 nm and a bandwidth of 2.14 nm.

2. We have a gain curve that can be described as: $\alpha(\nu) = \alpha_0 e^{-\frac{1}{2}\left(\frac{\nu-\nu_0}{\sigma}\right)^2}$, where $\sigma = 10^{12}$ Hz and the maximum absorbance of the 1 cm-long gain medium is 1. In this laser, the gain medium spans the entire cavity and has an index of refraction of 1.5. Losses (δ) are 0.2707.

(a). How many modes are oscillating?

(b). If we assume, for the sake of argument, that this is a gas laser at ∼300 K, at what temperature does the gas need to be such that only one mode is oscillating? Assume that α_0, L, n, and δ stay the same.

(c). Is changing temperature a good way to reduce the oscillating mode number? If not, how else can we get down to one oscillating mode?

3. We have a two-level system defined by two, orthogonal states $|+\rangle$ and $|-\rangle$. An operator, $\hat{\sigma}$, when acting on $|+\rangle$ has an eigenvalue of 1; when acting on $|-\rangle$, it has an eigenvalue of -1.

(a). Find the density operator form of $\hat{\sigma}$ (e.g., $|+\rangle\langle+| \pm |-\rangle\langle+| \pm \ldots$). Remember that individual elements of a density matrix can be calculated as: $\sigma_{mn} = \langle\phi_m|\hat{\sigma}|\phi_n\rangle$, where ϕ_m and ϕ_n are orthonormal basis vectors spanning the Hilbert space.

(b). Let's say we have a state: $|\theta\rangle = \sqrt{\frac{1}{2}}|+\rangle \; + \; \frac{e^{i\theta}}{\sqrt{2}}|-\rangle$ (for arbitrary θ). Find $\hat{\rho}_\theta$.

(c). Using $\hat{\rho}_\theta$ and $\hat{\sigma}$, find $\langle\sigma\rangle$.

(d). Is $|\theta\rangle$ a pure or a mixed state?

(e). What about the state represented by this operator: $\hat{\rho}_1 = \frac{1}{2}|+\rangle\langle+| + \frac{1}{2}|-\rangle\langle-|$. Is it a pure or mixed state?

4. I place a paramagnetic, spin-1/2 material inside a magnet at 1.5 K (129 μeV). At $t = 0$, I turn on a magnetic field until it reaches 10 T at $t = t_1$. From t_1 until t_2, I keep the field at 10 T, which produces a Zeeman splitting of 1158 μeV. At a time t_2, the field is instantaneously quenched to 0 T.
(a). Between times t_1 and t_2, are the spins in population inversion?
(b). Is there a way we can make a laser out of this Zeeman-split system?
(c). The spin relaxation for this material is defined by $T_1 = 1$ μs and $T_2 = 10$ ns. At what time after t_2 does the spin system return to equilibrium?