PHYS 4310, Homework 7 due on Wednesday October 28^{th}, 2015

Griffiths 3.13 (9 points), 3.25 (15 points), 3.28 (20 points)

For 3.13, please include a problem (d): show that $[A, BC] = [A, B]\,C + B\,[A, C]$.

For 3.28, do **not** find the expectation value of p^2. I'm only interested in your solution of $\phi_n(p,t)$ and your sketch of $|\phi_1(p,t)|^2$ and $|\phi_2(p,t)|^2$.

4. (16 points)
(1) For what range of ν is the function $f(x) = x^\nu$ in Hilbert space, on the interval $(0,1)$? Assume ν is real (can be positive or negative). Hint: Exploit the condition that we placed on a function to exist in a Hilbert space. 0^α is zero *only* for $\alpha > 0$ ($0^0 = 1$, *e.g.*). Also, you will need to handle the $\nu = -1/2$ case separately from the general solution of ν to make a determination on whether or not $\nu = -1/2$ should be included in the Hilbert space.
(2) For $\nu = -1/2$, is $f(x)$ in this Hilbert space? What about $xf(x)$ and $\left(\frac{d}{dx}f(x)\right)$? These should follow easily from your determination in (a).

5. (15 points) In the interaction picture (also known as the Dirac picture), we saw that: $|\Psi_I(t)\rangle = e^{iH_0t/\hbar}|\Psi_S(t)\rangle$ and $Q_I(t) = e^{iH_0t/\hbar}Q_S e^{-iH_0t/\hbar}$, where Q_S is an operator in the Schrödinger picture, Q_I in an operator in the interaction picture, and $H(t) = H_0 + H_I'$ is the time-dependent Hamiltonian. Using these definitions, show $H_I'|\Psi_I(t)\rangle = i\hbar\frac{d}{dt}|\Psi_I(t)\rangle$ and $\frac{dQ_I}{dt} = \frac{1}{i\hbar}[Q_I, H_0] + \frac{\partial Q_I}{\partial t}$. For one of these results, you will need to use the Schrödinger equation in ket form.