PHYS 5130, Homework Set 1, due at 5 pm on Thursday, Feb. 11$^{\text{th}}$.

1. To convert into momentum space, we can Fourier transform our 3D spatial wave-function, $\psi(\boldsymbol{r})$ using:

$$\phi(\boldsymbol{p}) \equiv \frac{1}{(2\pi\hbar)^{3/2}} \int e^{-i(\boldsymbol{p}\cdot\boldsymbol{r})/\hbar}\psi(\boldsymbol{r})\, d^3\boldsymbol{r}.$$

(a). (20 points) Using spherical coordinates and setting the polar axis along $\mathbf{p}$, find $\phi(\boldsymbol{p})$ for the ground state of hydrogen. Do the θ integral first.

(b). (10 points) Show that $\phi(\boldsymbol{p})$ is normalized. Sketch $\psi(\boldsymbol{r})$ versus $\mathbf{r}$ and $\phi(\boldsymbol{p})$ versus $\mathbf{p}$.

(c). (15 points) Calculate $\langle r^2 \rangle$ (in real space) and $\langle p^2 \rangle$ (in momentum space) for the ground state of hydrogen.

(d). (10 points) What is the expectation value of the kinetic energy in the ground state? Express your answer as a multiple of E_1 (-13.6 eV; the Rydberg constant).

2. Examine Fig. 1. Light polarized at 45 degrees, such that

$$\vec{E} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

is incident on a vertically-oriented linear polarizer and a quarter waveplate with its fast axis 45 degrees to the linear polarizer.
(a). (20 points) Write down the Jones vector representing the electric field at each of the numbered points in the diagram.

(b). (15 points) The quarter waveplate in Fig. 1(a) is now replaced with a photo-elastic modulator (PEM). The PEM provides a retardation as a function of time that varies between $\lambda/4$ and $-\lambda/4$. Sketch the intensity at (3) as a function of the PEM retardation modulation.

3. In class, we discussed how a static magnetic dipole moment, $\vec{\mu}_m(t) = \int_V \vec{r} \times \vec{J}_e(\vec{r}, t)\, dV$, could be non-zero for a single quantum state. Here:

$$\vec{J}_e(\vec{r}, t) = -\frac{Zei\hbar}{2m}\left(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi\right),$$

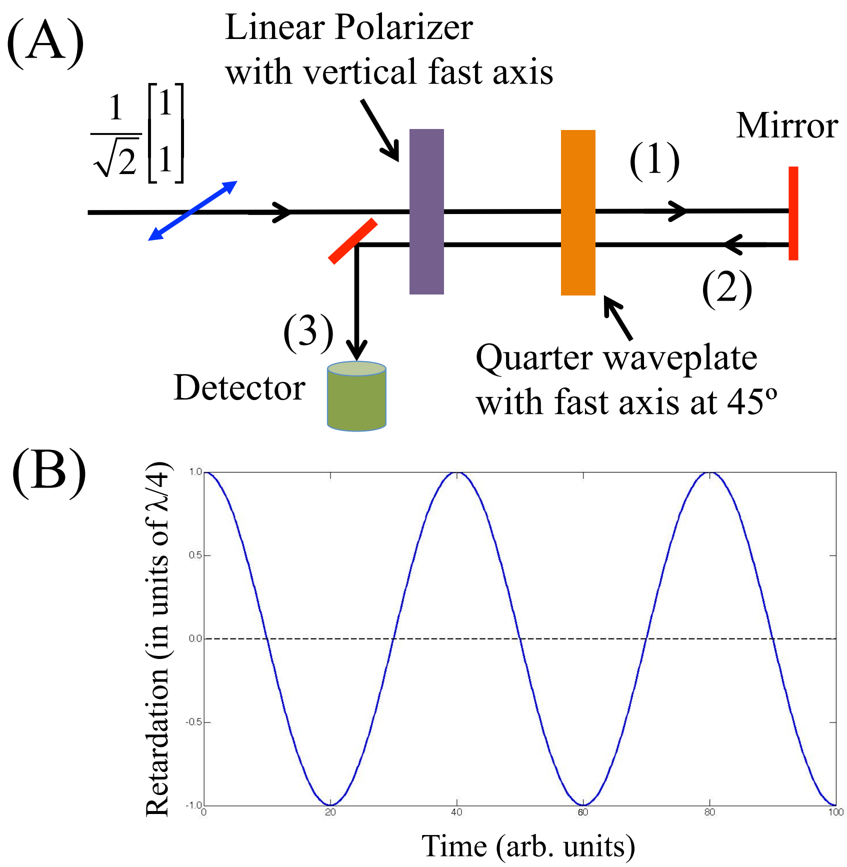

FIG. 1. (A) Faraday isolator set up. (B) Retardation, as a function of time, of the photo-elastic modulator (PEM). The PEM replaces the quarter wave plate in (A).

is the current density for an electron.
(a). (15 points) Find $\vec{\mu}_m(t)$ for Ψ_{211} of hydrogen.

(b). (10 points) Use your value in (a) to find $\langle\vec{\mu}_m(t)\rangle$.

(c). (20 points) Find $\vec{\mu}_m(t)$ for $\Psi = \frac{1}{\sqrt{5}}(\Psi_{210} + 2\Psi_{211})$ in hydrogen. This time-varying, or precessing, dipole moment can be probed using electron spin resonance.

4. (20 points) In the interaction picture (also known as the Dirac picture), we saw that: $|\Psi_I(t)\rangle = e^{iH_0t/\hbar}|\Psi_S(t)\rangle$ and $Q_I(t) = e^{iH_0t/\hbar}Q_S e^{-iH_0t/\hbar}$, where Q_S is an operator in the Schrödinger picture, Q_I in an operator in the interaction picture, and $H(t) = H_0 + H_I'$ is the time-dependent Hamiltonian. Using these definitions, show $H_I'|\Psi_I(t)\rangle = i\hbar\frac{d}{dt}|\Psi_I(t)\rangle$ and $\frac{dQ_I}{dt} = \frac{1}{i\hbar}[Q_I, H_0] + \frac{\partial Q_I}{\partial t}$. For one of these results, you will need to use the Schrödinger equation in ket form.