① $\sigma = 10^{-13}\ cm^2$

losses = 10%

$L = d = 15\ cm$

(a.) $\Delta N = \dfrac{r}{2\sigma(\nu)L} = \dfrac{0.1}{2(10^{-13}cm^2)(15\,cm)}$

$\boxed{\Delta N = 3.3\times10^{10}\ cm^{-3}}$

(b.) $P = 0.3\ mbar = 30\ Pa = 30\ N/m^2$

$PV = N k_B T$

$\dfrac{P}{k_B T} = \dfrac{N}{V}$

$\dfrac{N}{V} = \dfrac{30\ N/m^2}{1.381\times10^{-23} J/K (298\,K)} = 7.29\times10^{21}\ m^{-3} = 7.29\times10^{15}\ cm^{-3}$

$\dfrac{\Delta N}{N/V} = \dfrac{3.3\times10^{10}\ cm^{-3}}{7.29\times10^{15}\ cm^{-3}} = 4.57\times10^{-6}$ $\boxed{\simeq 5\times10^{-6}}$

② $\Delta\nu = \dfrac{c}{\lambda^2}\Delta\lambda = \dfrac{3\times10^{8}\ m/s}{(8\times10^{-7}m)^2}\ 10^{-9}\,m = 4.69\times10^{11}\ Hz$

$\alpha_{max} \simeq \dfrac{h\nu}{c\Delta\nu}(N_2-N_1)B_{12}$

$\dfrac{A_{21}}{B_{21}} = \dfrac{8\pi h\nu^3}{c^3} \longrightarrow$ if $B_{21}=B_{12}$, then $B_{12} = \dfrac{c^3}{8\pi h\nu^3}A_{21}$

$\therefore\ \alpha_{max} \simeq \dfrac{c^2}{\nu^2}\dfrac{N_2-N_1}{8\pi\Delta\nu}A_{21} = \dfrac{\lambda^2(N_2-N_1)}{8\pi\Delta\nu}A_{21}$

λ^2

$$\alpha_{max} \simeq \frac{(8\times10^{-5}cm)^2 (10^{18} cm^{-3})}{8\pi (4.69\times10^{11} Hz)} 10^4 s^{-1}$$ (where I have converted λ into cm)

$$\boxed{\alpha_{max} \simeq 5.4 \, cm^{-1}}$$

$A_{21} = 10^4 s^{-1} = \frac{1}{\text{lifetime}}$

3 (a.) $\lambda_L = 632.8$ nm $\rightarrow$ $\nu_L = \frac{c}{\lambda_L} = 4.74\times10^{14}$ Hz

$P_L = 10^{-3}$ W

$\Delta\nu_c = 4.7\times10^5$ Hz

$$\Delta\nu_L = \frac{2\pi h \nu_L (\Delta\nu_c)^2}{P_L} = \frac{2\pi \cdot 6.6\times10^{-34} J\text{-}s \times 4.74\times10^{14} Hz (4.7\times10^5 Hz)^2}{10^{-3} J/s}$$

$$\boxed{\Delta\nu_L = 4.3\times10^{-4} Hz}$$

Other books/sources leave off the 2 in the numerator so $\Delta\nu_L = 2.2\times10^{-4}$ Hz is acceptable as an answer, too.

(b.) $\lambda_L = 850$ nm $\longrightarrow$ $\nu_L = \frac{c}{\lambda_L} = 3.53\times10^{14}$ Hz

$P_L = 3\times10^{-3}$ W

$L = 300 \, \mu m$ $\longrightarrow$ $FSR = \frac{c}{2nL}$, where $\frac{c}{n}$ is the velocity in the cavity

$n = 3.5$

$$FSR = \frac{3\times10^8 m/s}{2(3.5)3\times10^{-4} m} = 1.43\times10^{11} Hz = 143 \text{ GHz}$$

$$\text{Finesse} = \frac{\pi\sqrt{R}}{1-R} = \frac{\pi(0.3)^{1/2}}{1-0.3} = 2.46$$

$$\Delta\nu_c = \frac{FSR}{\text{Finesse}} = \frac{143 \text{ GHz}}{2.46} = 58.1 \text{ GHz}$$

$$\Delta\nu_L = \frac{2\pi h \nu_L (\Delta\nu_c)^2}{P_L} = \frac{2\pi \times 6.6\times10^{-34}\,\text{J·s} \times 3.53\times10^{14}\,\text{Hz}\,(5.81\times10^{10}\,\text{Hz})^2}{3\times10^{-3}\,\text{W}}$$

$$= 1.65\times10^{6}\,\text{Hz} = \boxed{1.65\ \text{MHz}}$$

4 (a.) $\dfrac{\Delta\nu}{\nu} = \dfrac{\Delta d}{d} = \alpha\Delta T$

$\Delta T = 1°C = 1K$

$$\nu = \frac{c}{\lambda} = \frac{3\times10^{8}\,\text{m/s}}{5\times10^{-7}\,\text{m}} = 6\times10^{14}\,\text{Hz}$$

$$\Delta\nu = (6\times10^{14}\,\text{Hz})(1.2\times10^{-6}\,\text{K}^{-1})(1\,\text{K})$$

$$\boxed{\Delta\nu = 720\ \text{MHz}}$$

(b.) $\alpha = 4\times10^{-7}\,\text{K}^{-1}$, so $\boxed{\Delta\nu = 240\ \text{MHz}}$

This is three times better than invar.

5 $\delta\nu = 2\,\text{GHz}$

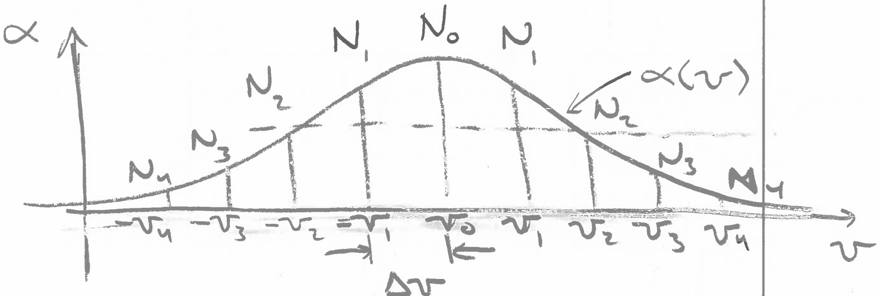
$\lambda_0 = 633\ \text{nm}$

$\Delta\nu = 50\ \text{MHz}$

$A_{ik} = 1\times10^{8}\,\text{s}^{-1}$

$L = 2(40\ \text{cm})$

losses = 10%



(a.) $$\Delta\nu = \frac{c}{2nd} = \frac{3\times10^{8}\,\text{m/s}}{2\times1\times0.4\,\text{m}} = \boxed{375\ \text{MHz}}$$

(b.) $$N(\nu) = N(\nu_0)\,e^{-(\nu-\nu_0)^2/\delta\nu^2}$$

$N(\nu_0)$ — population of N @ ν_0 [or N_0 for short]

$$N(\nu_1) = N_1 = N_0\,e^{-(\nu_1-\nu_0)^2/\delta\nu^2}$$

$$N_1 = N_0\,e^{-\left(\frac{375\,\text{MHz}}{2\,\text{GHz}}\right)^2} \Rightarrow N_1 = N_0\,e^{-(0.1875^2)} = \boxed{0.97N_0}$$

(c.) $$\Delta N_{threshold} = \frac{8\pi \Delta\nu}{2\lambda^2 L A_{ik}} r_{loss} = \frac{8\pi (5\times10^7\,\text{Hz})(0.1)}{2(6.33\times10^{-7}\,\text{m})^2(0.8\,\text{m})10^8\,\text{Hz}}$$

$$= \boxed{1.96\times10^{12}\ \text{m}^{-3}}$$

Above this threshold density, lasing occurs on a given mode. In this problem, we are assuming that the modes @ $\pm\nu_1$ are at this density.

(d.) $$\Delta N_1 = 0.97\,\Delta N_0$$

$$\frac{1.96\times10^{12}\ \text{m}^{-3}}{0.97} = \Delta N_0$$

$$\boxed{2.02\times10^{12}\ \text{m}^{-3} = \Delta N_0}$$