SUBJECT

DATE

$$\hat{H} = \alpha\left(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|\right)$$

$$\psi = c_1|1\rangle + c_2|2\rangle$$

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

$$\langle n|m\rangle = \delta_{nm}$$

$$\hat{H}|\psi\rangle = \alpha\left[|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|\right]$$

$$\left[c_1|1\rangle + c_2|2\rangle\right]$$

$$= \alpha\left[c_1|1\rangle + c_1|2\rangle - c_2|2\rangle + c_2|1\rangle\right] = E|\psi\rangle$$

$$\alpha(c_1 + c_2)|1\rangle + \alpha(c_1 - c_2)|2\rangle = E\left[c_1|1\rangle + c_2|2\rangle\right]$$

$$\alpha(c_1 + c_2) = Ec_1 \rightarrow c_2 = c_1\left(\frac{E}{\alpha} - 1\right)$$

$$\alpha(c_1 - c_2) = Ec_2 \rightarrow c_1 = c_2\left(\frac{E}{\alpha} + 1\right)$$

$$c_2 = \left(\frac{E}{\alpha} + 1\right)\left(\frac{E}{\alpha} - 1\right)c_2$$

$$1 = \frac{E^2}{\alpha^2} - 1 \rightarrow \boxed{E_\pm = \pm\alpha\sqrt{2}}$$

Eignvectors:

$$|\psi_\pm\rangle = c_1\left[|1\rangle + \underbrace{\left(\frac{E_\pm}{\alpha} - 1\right)}_{c_2}|2\rangle\right]$$

$$\boxed{|\psi_\pm\rangle = c_1\left[|1\rangle + (\pm\sqrt{2} - 1)|2\rangle\right]}$$

$$|1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \langle 1| = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad \langle 2| = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$|1\rangle\langle 1| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad |1\rangle\langle 2| = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$|2\rangle\langle 2| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad |2\rangle\langle 1| = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$\hat{H} = \alpha\left(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|\right)$$

$$\hat{H} = \alpha \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

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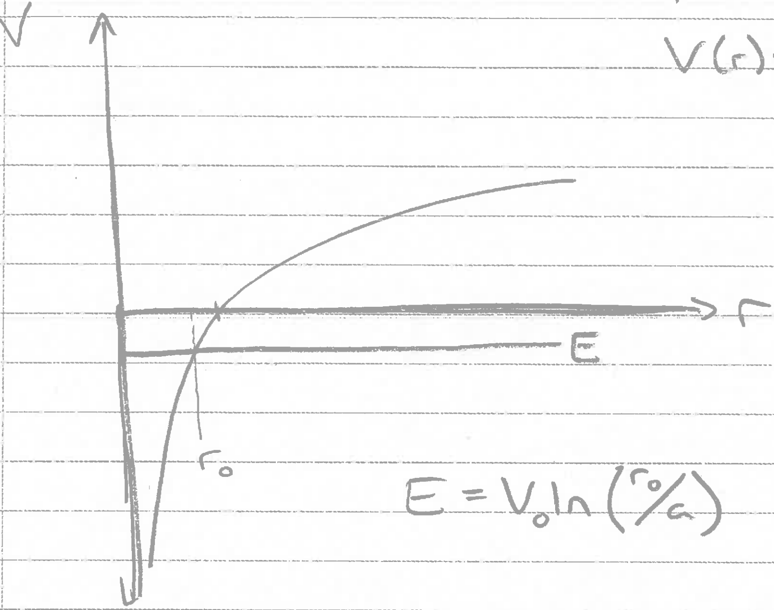
$$(n-\tfrac{1}{4})\pi\hbar = \int_0^{r_0} \underbrace{\sqrt{2m}\sqrt{E - V_0 \ln(r/a)}}_{p(r)}$$

$$V(r) = V_0 \ln(r/a)$$



$$E = V_0 \ln(r_0/a)$$

$$(n-\tfrac{1}{4})\pi\hbar = \sqrt{2m}\int_0^{r_0} \sqrt{V_0 \ln(r_0/a) - V_0 \ln(r/a)}\, dr$$

$$= \sqrt{2mV_0}\int_0^{r_0} \sqrt{\ln(r_0/r)}\, dr$$

$$x \equiv \ln(r_0/r) \rightarrow e^{x} = r_0/r \rightarrow r e^{x} = r_0 \rightarrow r = r_0 e^{-x}$$

$$dr = -r_0 e^{-x} dx$$

$$(n-\tfrac{1}{4})\pi\hbar = \sqrt{2mV_0}\int_{x_1}^{x_2} \sqrt{x}\, \underbrace{(-r_0 e^{-x})\, dx}_{dr}$$

$$\left.\begin{aligned} x_1 &= \ln(r_0/0) = \infty \\ x_2 &= \ln(r_0/r_0) = 0 \end{aligned}\right\} \quad (n-\tfrac{1}{4})\pi\hbar = -r_0\sqrt{2mV_0}\int_{\infty}^{0} \sqrt{x}\, e^{-x}\, dx$$

$$(n-1/4)\pi\hbar = r_0\sqrt{2mV_0}\int_0^\infty \sqrt{x}\, e^{-x}\,dx$$

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}\,dx\ , \quad \Gamma(1/2) = \sqrt{\pi}\ , \quad \Gamma(z+1) = z\Gamma(z)$$

$3/2 = 1 - 1/2$ $\quad 1/2$

$$(n-1/4)\pi\hbar = r_0\sqrt{2mV_0}\,\Gamma(3/2) = r_0\sqrt{2mV_0}\,\frac{\sqrt{\pi}}{2}$$

$$(n-1/4)\pi\hbar = r_0\sqrt{2mV_0}\,\frac{\sqrt{\pi}}{2}$$

$$\frac{2(n-1/4)\hbar\sqrt{\pi}}{\sqrt{2mV_0}} = r_0 \longrightarrow r_0 = \hbar(n-1/4)\sqrt{\frac{2\pi}{mV_0}}$$

$$E_n = V_0 \ln(r_0/a)$$

$$\boxed{E_n = V_0 \ln\left[\frac{\hbar}{a}\sqrt{\frac{2\pi}{mV_0}}(n-1/4)\right]} = V_0\left\{\ln(n-1/4) + \underline{\ln\left(\frac{\hbar}{a}\sqrt{\frac{2\pi}{mV_0}}\right)}\right\}$$

$$\ln\left(\frac{\hbar}{a}\sqrt{\frac{2\pi}{mV_0}}\right)$$

$$E_{n+1} - E_n = V_0\left\{\ln(n+1-1/4) - \ln(n-1/4)\right\}$$

$$= V_0 \ln\left(\frac{n+3/4}{n-1/4}\right)$$