HW5 Solns

1.

(a.)

$$E_{THz} = \frac{\lambda_{NIR}}{2\pi n_{NIR}^3 r_{41} l} \Gamma$$

$$= \frac{\cancel{8} \times 10^{\cancel{-7}\,-2} \text{ m}}{\cancel{2\pi}(2)^3 (3.9\times10^{-12} \text{ m/V})(\cancel{10^{-5}} \text{ m})} \frac{\cancel{\pi}}{6}$$

$$= \frac{10^{10} \text{ V/m}}{12\,(3.9)} = 2.14\times10^{8} \text{ V/m}$$

$$\boxed{= 2.14\times10^{6} \text{ V/cm}}$$

This max. field is incredibly strong for a THz pulse.

$$B_{THz} = \frac{E_{THz}}{c} = \frac{2.14\times\cancel{10^8} \text{ V/m}}{3\times\cancel{10^8} \text{ m/s}} = \boxed{0.712 \text{ T}}$$

(b.) For the pulse inside the semiconductor, we found $E_{peak} = 4.38\times10^{6}$ V/cm $= 4.38\times10^{8}$ V/m

$$B_{peak} = \frac{4.38\times10^{8} \text{ V/m}}{\underbrace{\frac{3\times10^{8} \text{ m/s}}{4}}_{c/n}} = \boxed{5.84 \text{ T}}$$

(assuming no absorption)

The index of refraction makes a tremendous difference in how large of a B-field the pulse "carries" w/ it. In comparison, the THz B-field is roughly an order of magnitude smaller than the B_{NIR} peak field in a semiconductor w/ $n = 4$.

(c.)

$\nu = 2\,\mathrm{THz}$

$h\nu = g\mu_B B_0$

$$\frac{h\nu}{g\mu_B} = B_0 \Rightarrow B_0 = \frac{6.626\times10^{-34}\,\mathrm{J\text{-}s} \times 2\times10^{12}\,\mathrm{Hz}}{2\times 9.274\times10^{-24}\,\mathrm{J/T}}$$

$$\boxed{B_0 = 71.4\,\mathrm{T}}$$

$\nu = \frac{3\times10^{8}\,\mathrm{m/s}}{800\times10^{-9}\,\mathrm{m}} = 3.75\times10^{14}\,\mathrm{Hz}$ (regardless of n, the light freq. does not Δ. The λ will, however.)

$$B_0 = \frac{6.626\times10^{-34}\,\mathrm{J\text{-}s}\times 3.75\times10^{14}\,\mathrm{Hz}}{2\times9.274\times10^{-24}\,\mathrm{J/T}}$$

$$\boxed{B_0 = 1.34\times10^{4}\,\mathrm{T}}$$

Thus, for both the THz and NIR pulses, the B_0 static field would have to be very high to produce a Zeeman splitting that will be resonant to the B_{pulse} @ these frequencies

[redacted]

2.

$$\mathcal{E}(t) = \mathcal{E}_0 \operatorname{sech}(t/\tau_s)$$

FWHM occurs when $\dfrac{\mathcal{E}(t)}{\mathcal{E}_0} = \dfrac{1}{2}$

$$\frac{1}{2} = \operatorname{sech}(t_0/\tau_s)$$

$$\operatorname{sech}^{-1}\left(\frac{1}{2}\right) = t_0/\tau_s$$

$$\tau_s \operatorname{sech}^{-1}\left(\frac{1}{2}\right) = t_0$$

$$\pm 1.317\,\tau_s = t_0$$

$$\boxed{\text{FWHM} = 2t_0 = 2.634\,\tau_s}$$

For the intensity profile, $I(t) \propto \mathcal{E}^2(t) = \mathcal{E}_0^2 \operatorname{sech}^2(t/\tau_s)$, we have:

$$\frac{1}{2} = \frac{\mathcal{E}^2(t)}{\mathcal{E}_0^2} = \operatorname{sech}^2(t_1/\tau_s)$$

$$\pm \tau_s \operatorname{sech}^{-1}\left(\frac{1}{\sqrt{2}}\right) = t_1$$

$$\pm 0.881\,\tau_s = t_1$$

$$\text{FWHM} = 2t_1 = 1.763\,\tau_s \quad [\text{for } I(t)]$$

3.

$$\frac{dS}{d\lambda} = 10^5$$

$$\lambda = 600\text{ nm}$$

$$\alpha = 30^\circ = \pi/6$$

$$d = \frac{\text{mm}}{1000\text{ grooves}} = 10^{-3}\text{ mm} = 10^{-6}\text{ m}$$

$$\frac{-D\lambda}{d^2\left[1-\left(\frac{\lambda}{d}-\sin\alpha\right)^2\right]^{3/2}} = \frac{dS}{d\lambda}$$

$$D = \left|-\frac{dS}{d\lambda}\,\frac{d^2\left[1-\left(\frac{\lambda}{d}-\sin\alpha\right)^2\right]^{3/2}}{\lambda}\right|$$

$$= 10^5\,\frac{(10^{-6}\text{ m})^2}{6\times10^{-7}\text{m}}\left[1-\left(\frac{6\times10^{-7}\text{m}}{10^{-6}\text{m}}-\sin\frac{\pi}{6}\right)^2\right]^{3/2}$$

$$\boxed{D = .164\text{ m} = 16.4\text{ cm}}$$

4.

(a.) Just inside the fiber, $n = 1.5$, so:

$$\frac{c}{n}\tau_G = \Delta x$$

$$\frac{3\times10^{8}\,\mathrm{m/s}}{1.5}\,5\times10^{-13}\,\mathrm{s} = \Delta x$$

$$\boxed{100\,\mu\mathrm{m} = \Delta x}$$

(b.)

$$\Delta n = \frac{dn}{d\lambda}\Delta\lambda \qquad \frac{dn}{d\lambda} = 10^{3}\,\mathrm{cm}^{-1}$$

$$\Delta\nu\,\tau_G = \frac{1}{2}$$

I will also accept 1

$$\Delta\nu = \frac{1}{2\tau_G}$$

$$\Delta\nu = \frac{1}{10^{-12}\,\mathrm{s}} = 10^{12}\,\mathrm{Hz}$$

$$\Delta\lambda = \frac{\lambda^2}{c}\Delta\nu$$

$$= \frac{(6\times10^{-7}\,\mathrm{m})^2}{3\times10^{8}\,\mathrm{m/s}}\,10^{12}\,\mathrm{Hz}$$

$= 1.2\times10^{-9}\,\mathrm{m} = 1.2\,\mathrm{nm}$ (if you used 1 for $\Delta\nu\tau_G$, then $\Delta\lambda = 2.4\,\mathrm{nm}$)

$$\Delta n = 10^{3}\,\mathrm{cm}^{-1}\times1.2\times10^{-7}\,\mathrm{cm} = 1.2\times10^{-4}$$

$$\Delta\tau_G = z_1\left(\frac{n+\Delta n}{c} - \frac{n}{c}\right) = z_1\frac{\Delta n}{c}$$

At z_1, $\Delta\tau_G = 500\,\mathrm{fs}$ (doubling of the pulsewidth):

$$z_1 = \frac{5\times10^{-13}\,\mathrm{s}\times3\times10^{8}\,\mathrm{m/s}}{1.2\times10^{-4}} = \boxed{1.25\,\mathrm{m}}$$

($0.625\,\mathrm{m}$ if $\Delta\nu\tau_G = 1$ was used)

5.

(a.) $$E_{at} = \frac{e}{4\pi \epsilon_0 a_0^2}$$

$$a_0 = 0.529\ \text{Å} = 5.29 \times 10^{-11}\ m$$

$$\epsilon_0 = 8.85 \times 10^{-12}\ F/m$$

$$e = 1.6 \times 10^{-19}\ C$$

$$E_{at} = \frac{1.6 \times 10^{-19}\ C}{4\pi (8.85 \times 10^{-12}\ F/m)(5.29 \times 10^{-11}\ m)^2}$$

$$= 5.14 \times 10^{11}\ V/m$$

$$\boxed{E_{at} = 5.14 \times 10^{9}\ V/cm}$$

(b.)

$$P^{(1)} \simeq P^{(2)} \simeq P^{(3)}$$

$$\cancel{\epsilon_0} \chi^{(1)} E_{at} \simeq \cancel{\epsilon_0} \chi^{(2)} E_{at}^2 \simeq \cancel{\epsilon_0} \chi^{(3)} E_{at}^3$$

if $\chi^{(1)}$ is taken to be order of 1, then:

$$\boxed{\chi^{(2)} \simeq \frac{1}{E_{at}} = 1.95 \times 10^{-12}\ m/V}$$

$$\boxed{\chi^{(3)} \simeq \frac{1}{E_{at}^2} = 3.78 \times 10^{-24}\ m^2/V^2}$$