PHYS 4310, Homework 11 due on Friday December 4$^{\text{th}}$, 2015

Griffiths (2$^{\text{nd}}$ edition): 4.19 (20 points), 4.20 (18 points), 4.21 (14 points), 4.22 (18 points)

Hints for 4.20: Remember your findings from problem 4.19 and the relation, $[f(x), p] = i\hbar \frac{df}{dx}$ [problem 3.13(c)$\Leftrightarrow$ equation 3.65].

4. [24 points] The probability current that we encountered earlier in the course was defined as $\mathbf{J} \equiv \frac{i\hbar}{2\mu}(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi)$, where μ is the reduced mass. Like electrical current, J is a conserved quantity obeying the continuity equation: $\nabla\cdot\mathbf{J} = -\frac{\partial}{\partial t}|\Psi|^2$. Due to the divergence theorem, $\int_S \mathbf{J}\cdot d\mathbf{a} = -\frac{d}{dt}\int_V |\Psi|^2 d^3\mathbf{r}$, where the LHS is integrated over the bounded surface, S, and the RHS is integrated over a fixed volume, V.

(a) Find $\mathbf{J}$ for $\psi_{2,1,1}$. Ans: $\frac{\hbar}{64\pi\mu a_0^5} r e^{-r/a_0}\sin\theta\hat{\phi}$.
(b) If we interpret $\mu\mathbf{J}$ as mass flow, we can write angular momentum, $\mathbf{L}$ as: $\mathbf{L} = \mu\int(\mathbf{r}\times\mathbf{J})\,d^3\mathbf{r}$. Use this definition of $\mathbf{L}$ to calculate out L_z for $\psi_{2,1,1}$. Comment on this result.