PHYS 4310 Study Guide for Mid-Term 2

Concepts that you should be very familiar with:
What is a Hilbert space
How to find the inner product of two functions
What does hermitian mean
Incompatible/compatible observables and the commutator
Eigenvalues/eigenfunctions and determinate states
Real space and momentum space
The generalized form of the uncertainty principle
Bra and ket notation
Separation of variables
Quantum numbers
Degeneracy
Bohr formula
Tunneling

Specific techniques that you should know how to use:
Continuity of wavefunctions
Conversion from real space to momentum space and vice versa
How to use the WKB method
How to use separation of variables
How to show that functions lie within the Hilbert space
How to use orthogonality and completeness to get "weighting coefficients"
How to scale the energy and wavefunctions of the hydrogen atom with reduced mass, atomic number, and dielectric constant

The Hamiltonian for a two-level system is:

$$\hat{H} = \alpha \left(|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1| \right), \tag{1}$$

where $|1\rangle$ and $|2\rangle$ are orthonormal basis vectors and α is some energy.
(a). Setup an eigenvalue equation using the generalized form of $|\psi\rangle$.
(b). Find the energy eigenvalues, E, in terms of α.
(c). Find the eigenfunctions, ψ_+ and ψ_- in terms of the basis vectors.
(d). What is $\hat{H}$, the Hamiltonian operator?

For spherically symmetrical potentials, like the $\frac{1}{r}$ one we used for the hydrogen atom, we can use the WKB approximation for the radial part (with $l = 0$) to solve for E_n. We can do this approximation by considering an infinite wall at $r = 0$:

$$\left(n - \frac{1}{4}\right)\pi\hbar = \int_0^{r_0} p\left(r\right) dr, \tag{2}$$

where $p\left(r\right)$ is the momentum as a function of the radial position. Let's consider a logarithmic potential, $V\left(r\right) = V_0 \ln\left(r/a\right)$, where V_0 and a are both constants.
(a). Sketch $V\left(r\right)$. Cast E in terms of r_0, where r_0 is the intersection of E and $V\left(r\right)$.
(b). Find E_n. Hint: change variables, $x \equiv \ln\left(r_0/r\right)$. $\Gamma\left(\alpha\right) = \int_0^\infty x^{\alpha-1}e^{-x}dx$, $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$, $\Gamma\left(\alpha+1\right) = \alpha\Gamma\left(\alpha\right)$.
(c). What is the energy spacing between $n+1$ and n?