PHYS 4310, Homework 9 due on Monday, November 18$^{\text{th}}$, 2015 at 5 pm

Griffiths (2$^{\text{nd}}$ edition): 4.18 (15 points)

2. (20 points) Using the Schrödinger equation for $l = 0$ and $V(r) = -\frac{Ze^2}{r}$ (cgs units) with $u = r\psi$ (for a hydrogen atom), substitute in an assumed form of $u(r)$: $u(r) = \left(Ar + Br^2\right) e^{-br}$. (*Hint*: use equation 4.53 for $l = 0$ and replace the V term with the one specified here.)
(a). Find the values of b and the ratio of B/A using this form of $u(r)$.
(b). Verify that it corresponds to the second energy level of hydrogen with E equal to $Z^2/4$ times the ground state of hydrogen and with $B/A = -Z/2a_0$ (where a_0 is the Bohr radius).
(c). What is the value of the coefficient b in terms of a_0?

3. (25 points) An important process in nuclear reactors and in nuclear weapons is the decay of tritium (^{3}H or H with two neutrons) into ionized ^{3}He via beta decay. This decay releases 18.6 keV per reaction with the ionized He going through another nuclear reaction (this time with a neutron) to produce ^{1}H and ^{3}H (or hydrogen and tritium). Consider the probability, P, that tritium will decay into ionized He by approximating the tritium wavefunction as the ground state of hydrogen and the ionized He wavefunction as the ground state of hydrogen with $Z = 2$.
(a). What is $P = |\langle\psi_{\text{He}}|\psi_{\text{T}}\rangle|^2$? I'm looking for both the explicit form of the integral and the computationally calculated value. *Hint*: P is between 0.5 and 1.0.
(b). What is P if the ψ_{He} is in l=1, $m = 0$ (excited state)?

4. (20 points) Consider a particle of mass m in a central potential:

$$V(r) = \frac{A}{r^2} + \frac{B}{r}, \tag{1}$$

where $A, B > 0$. Show that the bound states of angular momentum, l, exist with energies:

$$E_{n,l} = -\frac{mB^2}{2\hbar^2 (n + \lambda + 1)^2}, \tag{2}$$

for $n = 0, 1, 2, ...$ with $\lambda(\lambda + 1) = l(l + 1) + 2mA/\hbar^2$. Compare the degeneracy of this spectrum with that of the hydrogen atom ($A = 0$, $B = e^2$).