1.3 (a.) $\rho(x) = A\exp(-\lambda[x-a]^2)$

$$1 = \int_{-\infty}^{\infty} A\exp(-\lambda[x-a]^2)\,dx$$

$x - a = x'$

$dx = dx'$

$$1 = \int_{-\infty}^{\infty} A\exp(-\lambda x'^2)\,dx'$$

$$1 = \int_{-\infty}^{\infty} A\exp(-\lambda y'^2)\,dy'$$

$$1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A^2\exp(-\lambda[x'^2+y'^2])\,dx'dy'$$

$r^2 = x'^2 + y'^2$

$r\,dr\,d\theta = dx'dy'$

$$1 = A^2\int_0^{2\pi}\int_0^{\infty} r\exp(-\lambda r^2)\,d\theta\,dr$$

$z = \lambda r^2$

$dz = 2\lambda r\,dr$

$$\frac{1}{2\pi} = A^2\int_0^{\infty} e^{-z}\,\cancel{\frac{dz}{2\lambda}} \longrightarrow \frac{\lambda}{\pi} = A^2\left[-e^{-z}\Big|_0^{\infty}\right] = A^2[-0+1$$

$$\frac{\lambda}{\pi} = A^2$$

$$\boxed{\sqrt{\frac{\lambda}{\pi}} = A}$$

1.3 (b)

$$\rho(x) = \sqrt{\frac{\lambda}{\pi}}\, e^{-\lambda(x-a)^2}$$

$$\langle x\rangle = \int_{-\infty}^{\infty} x\rho(x)\,dx = \sqrt{\frac{\lambda}{\pi}}\int_{-\infty}^{\infty} x\, e^{-\lambda(x-a)^2}\,dx$$

$z = x-a$

$dz = dx$

$$\langle x\rangle = \sqrt{\frac{\lambda}{\pi}}\int_{-\infty}^{\infty}(z+a)\,e^{-\lambda z^2}\,dz$$

$$= \sqrt{\frac{\lambda}{\pi}}\left[\underbrace{\int_{-\infty}^{\infty} z e^{-\lambda z^2}\,dz}_{\text{since this is an odd function integrated over the entire x-axis, it will be 0. Let's show that:}} + a\underbrace{\int_{-\infty}^{\infty} e^{-\lambda z^2}\,dz}_{=\sqrt{\frac{\pi}{\lambda}}\ \{\text{we've already solved this integral in (a)}}\right]$$

$$\int_{-\infty}^{\infty} z e^{-\lambda z^2}\,dz = \int_{+\infty}^{\infty} e^{-u}\frac{du}{2\lambda}$$

$u = \lambda z^2$

$du = 2\lambda z\,dz$

$$= \frac{1}{2\lambda}\left[-e^{-u}\Big|_{+\infty}^{\infty}\right] = 0$$

$$\boxed{\langle x\rangle = a\sqrt{\frac{\lambda}{\pi}}\sqrt{\frac{\pi}{\lambda}} = a}$$

$$\langle x^2\rangle = \sqrt{\frac{\lambda}{\pi}}\int_{-\infty}^{\infty} x^2 \exp\left(-\lambda[x-a]^2\right)dx$$

$z = x-a$

$dz = dx$

$$= \sqrt{\frac{\lambda}{\pi}}\left[\underbrace{\int_{-\infty}^{\infty} z^2 e^{-\lambda z^2}\,dz}_{\text{we need a trick to solve this one:}} + 2a\underbrace{\int_{-\infty}^{\infty} z e^{-\lambda z^2}\,dz}_{=0} + a^2\underbrace{\int_{-\infty}^{\infty} e^{-\lambda z^2}\,dz}_{=\sqrt{\frac{\pi}{\lambda}}}\right]$$

If we differentiate the Gaussian integral wrt λ (not z), we get:

$$\frac{d}{d\lambda}\int_{-\infty}^{\infty} e^{-\lambda z^2}\,dz = -\int_{-\infty}^{\infty} z^2 e^{-\lambda z^2}\,dz$$

But we know the definite integral of $\int_{-\infty}^{\infty} e^{-\lambda z^2} dz$

$$\frac{d}{d\lambda}\left[\int_{-\infty}^{\infty} e^{-\lambda z^2} dz\right] = \frac{d}{d\lambda}\left[\sqrt{\frac{\pi}{\lambda}}\right] = \sqrt{\pi}\left[-\frac{1}{2}\lambda^{-3/2}\right]$$

$$\frac{d}{d\lambda}\left[\int_{-\infty}^{\infty} e^{-\lambda z^2} dz\right] = -\int_{-\infty}^{\infty} z^2 e^{-\lambda z^2} dz = -\frac{1}{2\lambda}\sqrt{\frac{\pi}{\lambda}}$$

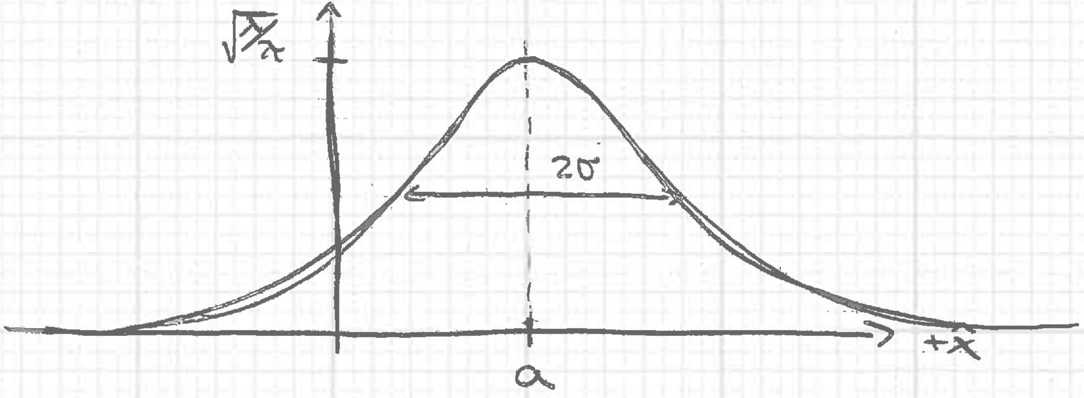
$$\therefore \int_{-\infty}^{\infty} z^2 e^{-\lambda z^2} dz = \boxed{\frac{1}{2\lambda}\sqrt{\frac{\pi}{\lambda}}}$$

As such, $\langle x^2 \rangle = \cancel{\sqrt{\frac{\lambda}{\pi}}}\left[\frac{1}{2\lambda}\cancel{\sqrt{\frac{\pi}{\lambda}}} + 0 + a^2\cancel{\sqrt{\frac{\pi}{\lambda}}}\right]$

$$\boxed{\langle x^2 \rangle = \frac{1}{2\lambda} + a^2}$$

$$\sigma = \left[\langle x^2 \rangle - \langle x \rangle^2\right]^{1/2} = \left[\left(\frac{1}{2\lambda} + a^2\right) - a^2\right]^{1/2} = \boxed{\sqrt{\frac{1}{2\lambda}}}$$

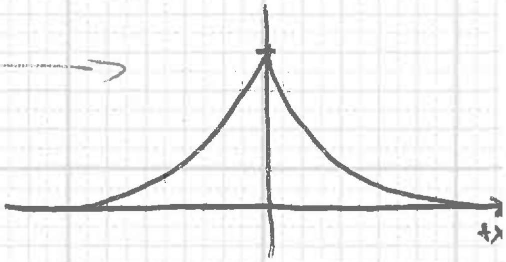
(c.) $\rho(x) = \sqrt{\frac{\lambda}{\pi}} \exp\left(-\lambda[x-a]^2\right)$



1.5 $\Psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t}$

(a.)

$$1 = \int_{-\infty}^{\infty}\Psi^*\Psi\,dx = A^2\int_{-\infty}^{\infty}e^{-2\lambda|x|}\,dx$$



B/c the function is even about 0, we can simplify our integral by removing the absolute value and taking our bnds of integration from 0 to ∞ (and multiplying by 2)

$$1 = 2A^2\int_0^\infty e^{-2\lambda x}\,dx \Rightarrow 1 = 2A^2\left[\frac{-1}{2\lambda}\left(\overset{1}{\cancel{e^{-2\lambda x}\Big|_0^\infty}}\right)\right]$$

$$\boxed{\sqrt{\lambda} = A}$$

(b.)

$$\langle x\rangle = \int_{-\infty}^{\infty}\Psi^* x\Psi\,dx = \lambda\underbrace{\int_{-\infty}^{\infty}xe^{-2\lambda|x|}\,dx}_{\text{odd function of } x \text{ means that integral} = 0.}$$

Let's show that:

$$= \lambda\left[\underbrace{\int_0^\infty xe^{-2\lambda x}\,dx}_{\text{positive \# integration}} + \underbrace{\int_0^\infty -xe^{-2\lambda x}\,dx}_{\text{integration over negative \#'s can be expressed this way}}\right] = \lambda\left[\cancel{\int_0^\infty xe^{-2\lambda x}\,dx - \int_0^\infty xe^{-2\lambda x}\,dx}\right] = 0$$

$$\boxed{\langle x\rangle = 0}$$

$$\langle x^2\rangle = \int_{-\infty}^{\infty}\Psi^* x^2\Psi\,dx = \lambda\underbrace{\int_{-\infty}^{\infty}x^2e^{-2\lambda|x|}\,dx}_{\text{even function, so let's take integral from 0 to } \infty\text{, remove the abs sign, and multiply by 2}}$$

$$\langle x^2\rangle = 2\lambda\int_0^\infty x^2 e^{-2\lambda x}\,dx \qquad u = 2\lambda x \quad du = 2\lambda\,dx$$

$$= 2\lambda\left[\frac{1}{8\lambda^3}\int_0^\infty u^2 e^{-u}\,du\right]$$

Using our IBP trick:

$$\begin{array}{ccc} u^2 & \searrow^{\oplus} & e^{-u} \\ 2u & \searrow^{\ominus} & -e^{-u} \\ 2 & \searrow^{\oplus} & e^{-u} \\ 0 & & -e^{-u} \end{array}$$

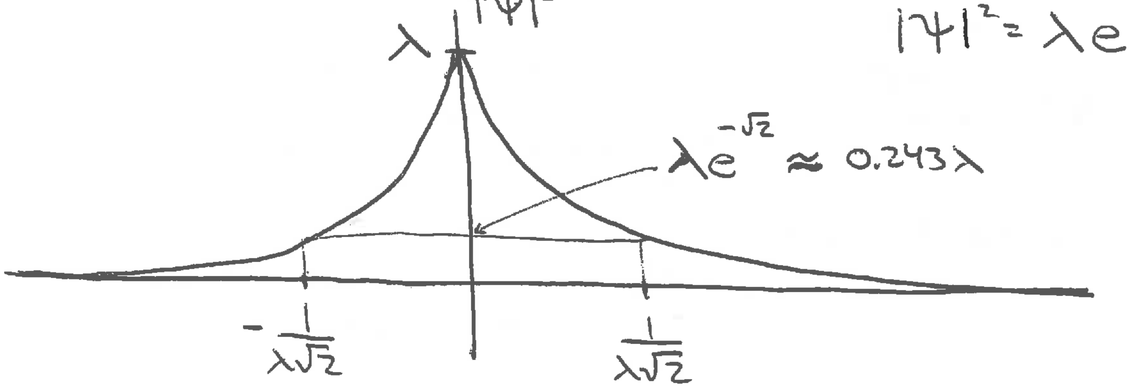
$$= \frac{1}{4\lambda^2}\left[-e^{-u}\left(u^2+2u+2\right)\Big|_0^\infty\right]$$

$$\boxed{\langle x^2\rangle = \frac{1}{2\lambda^2}}$$

(c) $$\sigma = \left[\langle x^2\rangle - \langle x\rangle^2\right]^{1/2}$$

$$\sigma = \left[\frac{1}{2\lambda^2} - 0\right]^{1/2} = \frac{1}{\lambda\sqrt{2}}$$

$$|\psi|^2 = \lambda e^{-2\lambda|x|}$$



Probability of particle will be found outside $x[-\sigma:+\sigma]$:

① $$P = 1 - \int_{-\sigma}^{\sigma}|\psi|^2\,dx = 1 - 2\int_0^{\sigma}\lambda e^{-2\lambda x}\,dx$$

$$= 1 - \left[\cancel{2\lambda}\left(-\frac{1}{\cancel{2\lambda}}\left\{e^{-\sqrt{2}} - 1\right\}\right)\right] = \cancel{1-1} + e^{-\sqrt{2}} = 0.243$$

② $$P = \int_{-\infty}^{-\sigma}|\psi|^2\,dx + \int_{\sigma}^{\infty}|\psi|^2\,dx = 2\int_{\sigma}^{\infty}\lambda e^{-2\lambda x}\,dx = \cancel{2\lambda}\left[-\frac{1}{\cancel{2\lambda}}\left(\underset{0}{\cancel{e^{-\infty}}} - e^{-\sqrt{2}}\right)\right] = e^{-\sqrt{2}} = 0.243$$

SUBJECT

DATE

1.7

$$\frac{d\langle p\rangle}{dt} = \frac{d}{dt}\left[-i\hbar\int\psi^*\frac{\partial\psi}{\partial x}dx\right] = -i\hbar\int\frac{\partial}{\partial t}\left(\psi^*\frac{\partial\psi}{\partial x}\right)dx$$

$$\frac{\partial}{\partial t}\left(\psi^*\frac{\partial\psi}{\partial x}\right) = \frac{\partial\psi^*}{\partial t}\frac{\partial\psi}{\partial x} + \psi^*\frac{\partial}{\partial t}\frac{\partial\psi}{\partial x} = \frac{\partial\psi^*}{\partial t}\frac{\partial\psi}{\partial x} + \psi^*\underbrace{\frac{\partial^2\psi}{\partial x\partial t}}_{\substack{\text{switched order of integration}\\ \text{(Fubini's thm)}}}$$

$$\frac{\partial\psi^*}{\partial t} = \frac{-i\hbar}{2m}\frac{\partial^2\psi^*}{\partial x^2} + \frac{i}{\hbar}V\psi^* \quad \text{(complex conjugate of SE)}$$

$$\frac{\partial\psi}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{i}{\hbar}V\psi \quad \text{(SE)}$$

$$\frac{\partial}{\partial t}\left(\psi^*\frac{\partial\psi}{\partial x}\right) = \frac{i\hbar}{2m}\underbrace{\left[\psi^*\frac{\partial^3\psi}{\partial x^3} - \frac{\partial^2\psi^*}{\partial x^2}\frac{\partial\psi}{\partial x}\right]}_{①} + \frac{i}{\hbar}\underbrace{\left[V\psi^*\frac{\partial\psi}{\partial x} - \psi^*\frac{\partial}{\partial x}(V\psi)\right]}_{②}$$

①
$$\int\Big(\underbrace{\psi^*\frac{\partial^3\psi}{\partial x^3}}_{Ⓐ} - \underbrace{\frac{\partial^2\psi^*}{\partial x^2}\frac{\partial\psi}{\partial x}}_{Ⓑ}\Big)dx = \underbrace{\psi^*\frac{\partial^2\psi}{\partial x^2}\Big|_{-\infty}^{\infty} - \int\frac{\partial\psi^*}{\partial x}\frac{\partial^2\psi}{\partial x^2}dx}_{\text{IBP of Ⓐ}}$$

$$\underbrace{-\frac{\partial\psi^*}{\partial x}\frac{\partial\psi}{\partial x}\Big|_{-\infty}^{\infty} + \int\frac{\partial\psi^*}{\partial x}\frac{\partial^2\psi}{\partial x^2}dx}_{\text{IBP of Ⓑ}}$$

0

0

cancel

$$= 0$$

②
$$\int\left[V\psi^*\frac{\partial\psi}{\partial x} - \psi^*\frac{\partial}{\partial x}(V\psi)\right]dx = \int V\psi^*\frac{\partial\psi}{\partial x}dx - \int\psi^*\frac{\partial V}{\partial x}\psi\,dx$$

$$-\int\psi^* V\frac{\partial\psi}{\partial x}dx$$

cancel

$$= -\int\psi^*\frac{\partial V}{\partial x}\psi\,dx = \left\langle-\frac{\partial V}{\partial x}\right\rangle$$

Putting this altogether:

$$\frac{d\langle p\rangle}{dt} = -\cancel{i\hbar}\left(\frac{i}{\cancel{\hbar}}\left\langle-\frac{\partial V}{\partial x}\right\rangle\right) = \left\langle-\frac{\partial V}{\partial x}\right\rangle \quad \text{QED}$$

1.9

$$\Psi(x,t) = A\exp\left[-a\left(\frac{mx^2}{\hbar}+it\right)\right]$$

(a.)

$$1 = \int \Psi^*\Psi\,dx = A^2\underbrace{\int \exp\left(-\frac{2amx^2}{\hbar}\right)dx}$$

we've already solved this integral in 1.3. For $\lambda = \frac{2am}{\hbar}$, this integral is equal to ~~[illegible]~~ $\left(\frac{\pi\hbar}{2am}\right)^{1/2}$, where $\int_{-\infty} e^{-\lambda x^2}\,dx = 1$

$$1 = A^2\left(\frac{\pi\hbar}{2am}\right)^{1/2}$$

$$\boxed{\left(\frac{2am}{\pi\hbar}\right)^{1/4} = A}$$

(b.)

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V\Psi$$

$\Psi = A\psi(x)\varphi(t)$, where $\psi(x) = \exp\left(-\frac{am}{\hbar}x^2\right)$; $\varphi(t)=\exp(-iat)$

LHS of the SE: $i\hbar\frac{\partial\Psi}{\partial t} = i\hbar A\psi(x)\frac{\partial\varphi}{\partial t} \Rightarrow i\hbar A\psi(x)(-ia)\varphi(t)$

$$= a\hbar\Psi(x,t)$$

RHS of the SE:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} = -\frac{\hbar^2}{2m}A\varphi(t)\frac{\partial^2\psi(x)}{\partial x^2} \Rightarrow \frac{\partial}{\partial x} = -\frac{2am}{\hbar}x\,\psi(x)$$

1st term only

$$\frac{\partial^2}{\partial x^2} = \left(\frac{-2am}{\hbar}+\frac{4a^2m^2}{\hbar^2}x^2\right)\psi(x)$$

$$= -\frac{\hbar^2}{2m}A\varphi(t)\left[-\frac{2am}{\hbar}+\frac{4a^2m^2}{\hbar^2}x^2\right]\psi(x)$$

$$= \left(a\hbar - 2a^2mx^2\right)\Psi(x,t)$$

Putting it together:

$$a\hbar\cancel{\Psi(x,t)} = \left(a\hbar-2a^2mx^2\right)\cancel{\Psi(x,t)} + V\cancel{\Psi(x,t)}$$

$$\cancel{a\hbar} = \cancel{a\hbar} - 2a^2mx^2 + V(x)$$

$$\boxed{2a^2mx^2 = V(x)}$$

1.9 (c)

$$\langle x \rangle = \int \Psi^* x \Psi \, dx = \underbrace{\int_{-\infty}^{\infty} A^2 x e^{-2am x^2/\hbar} \, dx}_{} = 0$$

odd function integrated over a symmetric interval about x=0. Must equal to 0.

$$\langle x^2 \rangle = \int \Psi^* x^2 \Psi \, dx = A^2 \underbrace{\int_{-\infty}^{\infty} x^2 e^{-\frac{2mx^2a}{\hbar}} \, dx}_{}$$

$$\lambda = \frac{2am}{\hbar} \longrightarrow \int_{-\infty}^{\infty} x^2 e^{-\lambda x^2} \, dx = \frac{1}{2\lambda}\sqrt{\frac{\pi}{\lambda}}$$

(see 1.3(b) for how to do this integral)

$$\langle x^2 \rangle = A^2 \frac{\hbar}{4am}\sqrt{\frac{\pi\hbar}{2am}} = \sqrt{\frac{2am}{\pi\hbar}}\frac{\hbar}{4am}\sqrt{\frac{\pi\hbar}{2am}} = \boxed{\frac{\hbar}{4am}}$$

$$\langle p \rangle = \frac{\hbar}{i}\int A^2 \Psi^*(x) \frac{\partial}{\partial x}\Psi(x) \, dx = -i\hbar \int \Psi^* \left(-\frac{2am}{\hbar} x\right) \Psi \, dx$$

$$= A^2 \frac{2am}{i} \underbrace{\int_{-\infty}^{\infty} x e^{-\frac{2am}{\hbar}x^2} \, dx}_{} = 0$$

odd function integrated over a symmetric interval about 0

$$\langle p^2 \rangle = \int \Psi^* \left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)^2 \Psi \, dx = -A^2\hbar^2 \int \Psi^*(x) \underbrace{\left[-\frac{2am}{\hbar} + \frac{4a^2m^2}{\hbar^2}x^2\right]}_{\text{from 1.9(b)}} \Psi(x) \, dx$$

$$= -A^2\hbar^2 \left[\int_{-\infty}^{\infty} -\frac{2am}{\hbar} e^{-\frac{2am}{\hbar}x^2} \, dx + \int_{-\infty}^{\infty} \frac{4a^2m^2}{\hbar^2} x^2 e^{-\frac{2am}{\hbar}x^2} \, dx\right]$$

$$= A^2\hbar^2 \left[\frac{2am}{\hbar} \underbrace{\int_{-\infty}^{\infty} e^{-\frac{2am}{\hbar}x^2} \, dx}_{\sqrt{\frac{\pi\hbar}{2am}} \text{ (see 1.9(a))}} - \int_{-\infty}^{\infty} \left(\frac{4a^2m^2}{\hbar^2}\right) \underbrace{x^2 e^{-\frac{2am}{\hbar}x^2} \, dx}_{\frac{\hbar}{4am}\sqrt{\frac{\pi\hbar}{2am}}}\right]$$

(we just did this integral above)

⑨

$$\langle p^2\rangle = A^2\hbar^2\left[\frac{2am}{\hbar}\sqrt{\frac{\pi\hbar}{2am}} - \left(\frac{\cancel{4}a^2m^2}{\hbar^2}\right)\frac{\cancel{\hbar}}{\cancel{4am}}\sqrt{\frac{\pi\hbar}{2am}}\right]$$

$$= \sqrt{\frac{2am}{\pi\hbar}}\sqrt{\frac{\pi\hbar}{2am}}\left[2am\hbar - am\hbar\right] = \boxed{am\hbar}$$

(d.)

$$\sigma_x = \left[\langle x^2\rangle - \langle x\rangle^2\right]^{1/2} \boxed{= \sqrt{\frac{\hbar}{4am}}}$$

$$\sigma_p = \left[\langle p^2\rangle - \langle p\rangle^2\right]^{1/2} \boxed{= \sqrt{am\hbar}}$$

$$\boxed{\sigma_x\sigma_p = \frac{\hbar}{2}}$$

This is consistent (to the maximal limit) w/ the uncertainty principle.

1.14

(a.) $P_{ab}(t) = \int_a^b |\Psi(x,t)| dx$

$$\frac{dP_{ab}(t)}{dt} = \frac{d}{dt}\int_a^b |\Psi|^2 dx = \int_a^b \underbrace{\frac{\partial}{\partial t}|\Psi|^2 dx}$$

see Eq. 1.25 or the lecture notes

$$\frac{dP_{ab}}{dt} = -\int_a^b \frac{\partial}{\partial x} J(x,t)\, dx \qquad \frac{\partial}{\partial x}\underbrace{\left[\frac{i\hbar}{2m}\left(\Psi^*\frac{\partial\Psi}{\partial x} - \frac{\partial\Psi^*}{\partial x}\Psi\right)\right]}_{-J(x,t)}$$

$$\boxed{\frac{dP_{ab}}{dt} = -J(x,t)\Big|_a^b = J(a,t) - J(b,t)}$$ QED

P_{ab} is dimensionless, so $\left[\frac{dP_{ab}}{dt}\right] = \boxed{\frac{1}{sec} \text{ or } sec^{-1}}$

(b.)

$$J(x,t) = \frac{i\hbar}{2m}\left[\Psi\frac{\partial\Psi^*}{\partial x} - \Psi^*\frac{\partial\Psi}{\partial x}\right]$$

$\Psi(x,t) = \psi(x)\phi(t)$, where $\psi(x) = Ae^{-\frac{am}{\hbar}x^2}$, $\phi(t) = e^{-iat}$

$$\frac{\partial\Psi^*}{\partial x} = -\frac{2am}{\hbar}x\,\psi(x)\phi^*(t)$$

$$\frac{\partial\Psi}{\partial x} = -\frac{2am}{\hbar}x\,\psi(x)\phi(t)$$

$$J(x,t) = -\frac{i\hbar}{2m}\left[\left(-\frac{2am}{\hbar}x\right)\psi^2(x) - \left(-\frac{2am}{\hbar}x\right)\psi^2(x)\right] = \boxed{0}$$

1.18

(a.)

$$\lambda = \frac{h}{\sqrt{3mk_BT}} \longrightarrow 3mk_BT = \frac{h^2}{\lambda^2}$$

$$T = \frac{h^2}{\lambda^2}\frac{1}{3mk_B}$$

$m_e = 9.1\times10^{-31}$ kg $\quad m_{Na} \approx \frac{.023\text{ kg}}{6.022\times10^{23}} = 3.82\times10^{-26}$ kg

$\lambda \geq d = 3Å = 3\times10^{-10}$ m

$k_B = 1.381\times10^{-23}\ \frac{m^2\text{-}kg}{s^2\text{-}K}$

$h = 6.626\times10^{-34}$ J-s

$$T_e \leq \frac{(6.626\times10^{-34}\text{ J-s})^2}{3(9\times10^{-20}\text{ m}^2)(9.1\times10^{-31}\text{ kg})(1.381\times10^{-23}\ \frac{m^2\text{-}kg}{s^2\text{-}K})}$$

$\boxed{T_e \leq 1.28\times10^5\text{ K}}$ ⟶ $e^{\ominus}$'s are pretty much always QM

$$T_{Na} \leq \frac{(6.626\times10^{-34}\text{ J-s})^2}{3(9\times10^{-20}\text{ m}^2)(3.82\times10^{-26}\text{ kg})(1.381\times10^{-23}\ \frac{m^2\text{-}kg}{s^2\text{-}K})}$$

$\boxed{T_{Na} \leq 3.09\text{ K}}$ only @ low T do the nuclei behave in a QM fashion

(b.) $PV = Nk_BT$

$\frac{P}{k_BT} = \frac{N}{V} \Rightarrow$ density $= \frac{\text{particles}}{\text{unit volume}}$

$V = \frac{4}{3}\pi R^3$

$\frac{V}{N} = V$ (for $N=1$) $= \frac{k_BT}{P}$

$$\frac{4\pi R^3}{3} = \frac{k_BT}{P}$$

$$R = \left(\frac{3k_BT}{4\pi P}\right)^{1/3}$$

$$T_{IG} \leq \frac{h^2}{3\left[\frac{3k_BT}{4\pi P}\right]^{2/3} mk_B}$$

$$\leq \frac{h^2}{3mk_B}\frac{(4\pi P)^{2/3}}{(3k_BT)^{2/3}}$$

~~$\leq \frac{h^2}{m}\frac{(4\pi P)^{2/3}}{(3k_B)^{5/3}}$~~

$$T_{IG}^{5/3} \leq \frac{h^2}{3mk_B}\left(\frac{4\pi P}{3k_B}\right)^{2/3}$$

$$T_{IG}^{5/3} \leq \frac{h^2}{m}(4\pi P)^{2/3}(3k_B)^{-5/3}$$

$$T_{IG} \leq \left(\frac{h^2}{m}\right)^{3/5}(4\pi P)^{2/5}(3k$$

$$T_{IG} \le \frac{1}{3k_B}\left(\frac{h^2}{m}\right)^{3/5}\left(4\pi P\right)^{2/5}$$

The book gives a different answer since they assume that $\frac{V}{N} \doteq \underset{\nearrow\ (\text{for } N=1)}{V} = d^3$. This is not the typical approximation. However, let's solve for T_{IG}.

$$d^3 = \frac{k_B T}{P} \rightarrow d = \left(\frac{k_B T}{P}\right)^{1/3}$$

$$T_{IG} \le \frac{h^2}{3mk_B\cancel{T}\left(\frac{k_B T}{P}\right)^{2/3}}$$

$$T_{IG}^{5/3} \le \frac{h^2 P^{2/3}}{3m\ k_B^{5/3}}$$

$$T_{IG} \le \left(\frac{h^2}{3m}\right)^{3/5}\left(\frac{P^{2/5}}{k_B}\right) = \frac{1}{k_B}\left(\frac{h^2}{3m}\right)^{3/5} P^{2/5}$$

He @ 1 atm

$$h = 6.626\times10^{-34}\ \text{J-s}\ ;\ k_B = 1.38\times10^{-23}\ \frac{\text{m}^2\text{-kg}}{\text{s}^2\text{-K}}$$

$$m_{He} = \frac{4\times10^{-3}\ \text{kg}}{6.022\times10^{23}} = 6.64\times10^{-27}\ \text{kg}$$

$$P = 1\ \text{atm} = 1.01\times10^5\ \text{Pa} = 1.01\times10^5\ \text{N/m}^2$$

$$T_{IG} \le \frac{1}{1.38\times10^{-23}\ \frac{\text{m}^2\text{-kg}}{\text{s}^2\text{-K}}}\left(\frac{[6.626\times10^{-34}\ \text{J-s}]^2}{3\cdot6.64\times10^{-27}\ \text{kg}}\right)^{3/5}\left(1.01\times10^5\ \text{N/m}^2\right)^{2/5}$$

$$\boxed{T_{IG} \le 2.9\ \text{K}}$$

H @ $T = 3\text{K}$, $d = 10^{-2}\ \text{m}$

①

$$\lambda = \frac{h}{\sqrt{3m_H k_B T}} = \frac{6.626\times10^{-34}\ J\text{-}s}{\sqrt{3\times1.7\times10^{-27}\ \cancel{kg}\times1.38\times10^{-23}\ \frac{m^2\text{-}\cancel{kg}}{s^2\text{-}K}\times3K}}$$

$= \boxed{1.5\ nm}$ which is much less than the inter-particle spacing of 1 cm ⇒ not QM

②

$$T \leq \frac{h^2}{\lambda^2}\frac{1}{3mk_B} = \frac{(6.626\times10^{-34}\ J\text{-}s)^2}{(10^{-2}m)^2(3\times1.7\times10^{-27}\ kg\times1.38\times10^{-23}\ \frac{m^2\text{-}\cancel{kg}}{s^2\text{-}kg})}$$

$= \boxed{6.4\times10^{-14}\ K}$ which is much less than 3K. As w/ ①, H is outer space cannot be treated quantum mechanically.

7.

$$\underline{\underline{M}} = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 0 \\ 5 & 0 & 3 \end{bmatrix}$$

$$\det \begin{vmatrix} 1-\lambda & 2 & 4 \\ 2 & 3-\lambda & 0 \\ 5 & 0 & 3-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(3-\lambda)^2 + 0 + 0 - 20(3-\lambda) - 4(3-\lambda) = 0$$

$$(3-\lambda)\left[(1-\lambda)(3-\lambda) - 24\right] = 0$$

$$\boxed{\lambda = 3}$$

$$\lambda^2 - 4\lambda - 21 = 0$$

$$(\lambda-7)(\lambda+3) = 0$$

$$\boxed{\lambda = 7, -3}$$

$\lambda = 3$

$$\begin{bmatrix} \overset{1}{\cancel{-2}} & \overset{1}{\cancel{2}} & \overset{2}{\cancel{4}} \\ 2 & 0 & 0 \\ 5 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-v_1 + v_2 + 2v_3 = 0$$

$$2v_1 = 0 \longrightarrow v_1 = 0$$

$$v_2 = -2v_3 \Rightarrow \text{if } v_3 = 1 \text{, then } v_2 = -2$$

$$v_1 = \sqrt{\frac{1}{5}} \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

$\lambda = -3$

$$\begin{bmatrix} \cancel{4}^{2} & \cancel{2}^{1} & \overset{2}{\cancel{4}} \\ \cancel{2}^{1} & \cancel{6}^{3} & 0 \\ 5 & 0 & 6 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \begin{matrix} 2v_1 + v_2 + 2v_3 = 0 & ① \\ v_1 + 3v_2 + 0 = 0 & ② \\ 5v_1 + 6v_3 = 0 & ③ \end{matrix}$$

From ②   From ③

$V_1 = -3V_2$ $v_1 = -\frac{6}{5} v_3$

If $v_3 = 5$, then $v_1 = -6$ and $v_2 = 2$

$$v_2 = \begin{bmatrix} -6 \\ 2 \\ 5 \end{bmatrix} \xrightarrow{\text{normalizing}} \sqrt{\frac{1}{65}} \begin{bmatrix} -6 \\ 2 \\ 5 \end{bmatrix}$$

$\lambda = 7$

$$\begin{bmatrix} \overset{-3}{\cancel{-6}} & \overset{1}{\cancel{2}} & \overset{2}{\cancel{4}} \\ \overset{1}{\cancel{2}} & \overset{-2}{\cancel{-4}} & 0 \\ 5 & 0 & -4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-3v_1 + v_2 + 2v_3 = 0$

$v_1 - 2v_2 = 0 \longrightarrow v_1 = 2v_2$

$5v_1 - 4v_3 = 0 \longrightarrow v_1 = \frac{4}{5} v_3$

If $v_3 = 5$, then $v_1 = 4$ and $v_2 = 2$

$$v_3 = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix} \xrightarrow{\text{normalizing}} \frac{1}{3\sqrt{5}} \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

$$\langle v_1 | v_2 \rangle = \frac{1}{\sqrt{5}} \begin{bmatrix} 0 & -2 & 1 \end{bmatrix} \frac{1}{\sqrt{65}} \begin{bmatrix} -6 \\ 2 \\ 5 \end{bmatrix} = \frac{1}{5\sqrt{13}} (0 - 4 + 5) = \frac{-1}{5\sqrt{13}}$$

$$\langle v_1 | v_3 \rangle = \frac{1}{\sqrt{5}} \begin{bmatrix} 0 & -2 & 1 \end{bmatrix} \frac{1}{3\sqrt{5}} \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix} = \frac{1}{15} (0 - 4 + 5) = \frac{-1}{15}$$

$$\langle v_2 | v_3 \rangle = \frac{1}{\sqrt{65}} \begin{bmatrix} -6 & 2 & 5 \end{bmatrix} \frac{1}{3\sqrt{5}} \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix} = \frac{1}{15\sqrt{13}} (-24 + 4 + 25) = \frac{1}{3\sqrt{13}}$$

None of the eigenvectors are orthogonal to one another.