PHYS 5130, Final Exam

Potentially helpful constants and conversions: 1 fs = 10^{-15} s; $c = 3\times10^8$ m/s; $h = 6.626\times10^{-34}$ J-s; $\hbar$=1.055×10^{-34} J-s; $h\nu = \frac{hc}{\lambda} = \frac{1240 \text{ eV}-\text{nm}}{\lambda}$; $\mu_B = 9.274\times10^{-24}$ J/T; 1 eV = 1.603×10^{-19} J; $e^{i\theta} + e^{-i\theta} = 2\cos\theta$.

1. **[35 points]** Up-chirped pulse.
(a) [5 points] Sketch an up-chirped pulse (envelope and actual E-field) with a Gaussian form.
(b) [10 points] Assume a center frequency of ω_0 and a chirp parameter a. If the pulse is moving in the positive $\hat{z}$ direction and is linearly polarized along with $\hat{x}$ direction, what is the complete mathematical description of this pulse?
(c) [20 points] What is $I(t)$ for this pulse? From the form of $I(t)$, what is $\omega(t)$, the angular frequency of the carrier wave?

2. **[30 points]** Pulse calculations.
(a) [13 points] As we saw in Homework 5, the characteristic atomic electric field is $\sim 5\times 10^9$ V/cm. What pulse width (assume a Gaussian pulse) is necessary to obtain this value of the field assuming a 1 kHz repetition rate laser with a power of 2 W is focused to a beam waist of 10 μm?
(b) [10 points] Using your answer from (a) for τ_G, if the center wavelength is 800 nm, how broad is its spectral width (in nm)?
(c) [7 points] How many photons are there in this pulse?

3. **[40 points]** We have a HeNe laser (λ=632.80 nm) with an output power of 10 mW. The index of refraction of the cavity is estimated at 1.25, and the resonator cavity is 40 cm long with a mirror reflectivity of 0.99.
(a) [10 points] What is the cavity FWHM, $\Delta\nu_c$, for this system?
(b) [10 points] What is the smallest single-mode laser linewidth, $\Delta\nu_L$, we can achieve for these conditions?
(c) [20 points] Assume we have a CCD camera with 20 μm pixels situated 150 mm away from our 2000 groove/mm grating. Using the grating equation for first-order reflections with the incident light at a 30° angle, $\lambda = d\,[0.5 + \sin\theta_r]$, what is the minimum bandwidth, $\Delta\lambda$, about 632.80 nm that we can disperse beyond one pixel width? Hints: Calculate the minimum dispersion necessary for half the bandwidth: that is, from 632.80 nm to 632.80+$\delta\lambda$, where $\Delta\lambda = 2 * \delta\lambda$. The bandwidth is going to be small (> 0.1 nm), so make sure to carry your significant figures out to at least four decimal places.

4. **[20 points]** We successfully generated a 5 fs Gaussian pulse centered at 500 nm.
(a) [5 points] How many optical cycles are in this pulse?
(b) [15 points] Does the slowly-varying envelope approximation apply in this situation? Why or why not? Recall that one of the forms that the SVEA could be written was: $|\frac{d}{dt}\tilde{\mathcal{E}}(t)| \ll \omega_0 |\tilde{\mathcal{E}}(t)|$, where ω_0 is the carrier frequency and $\tilde{\mathcal{E}}(t)$ is the field envelope.