Home Work #4 Solutions

1.

(a.) (+5 points)

$$f(x) = e^{-ax^2}$$

$\frac{1}{2\pi}$ is also fine

$$F(t) = \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} e^{-ax^2} e^{ixt}\, dx$$

$$-ax^2 + itx \rightarrow -a\left(x^2 - \frac{it}{a}x\right) \rightarrow -a\underbrace{\left[x^2 - \frac{it}{a}x - \frac{t^2}{4a^2}\right]}_{\left(x - \frac{it}{2a}\right)^2} - \frac{t^2}{4a}$$

$$F(t) = \frac{e^{-t^2/4a}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-a\left(x - \frac{it}{2a}\right)^2} dx$$

$$y = x - \frac{it}{2a}$$

$$dy = dx$$

$$F(t) = \frac{e^{-t^2/4a}}{\sqrt{2\pi}} \underbrace{\int_{-\infty}^{\infty} e^{-ay^2}\, dy}_{\sqrt{\frac{\pi}{a}}} = \boxed{\frac{1}{\sqrt{2a}} e^{-t^2/4a}}$$

(b.) $f(v) = A e^{-\frac{(v - v_0)^2}{0.366\,\delta v_D^2}}$ (+5 points)

$$\delta v_D = 8 \times 10^9 \text{ s}^{-1}$$

In analogy w/ (a.), $a = \frac{1}{0.366\,\delta v_D^2}$ and $v_0 = 0$.

FWHM occurs when $e^{-t^2/4a} = \frac{1}{2} \Rightarrow \ln 2 = t^2/4a$

$$\pm 2\sqrt{a \ln 2} = t$$

So: t_p = FWHM in the time domain $= 4\sqrt{a\ln 2}$

$$t_p = 4\sqrt{\frac{\ln 2}{0.366\,\delta\nu_D^2}} = \frac{4}{8\times10^9\,\text{Hz}}\sqrt{\frac{\ln 2}{0.366}}$$

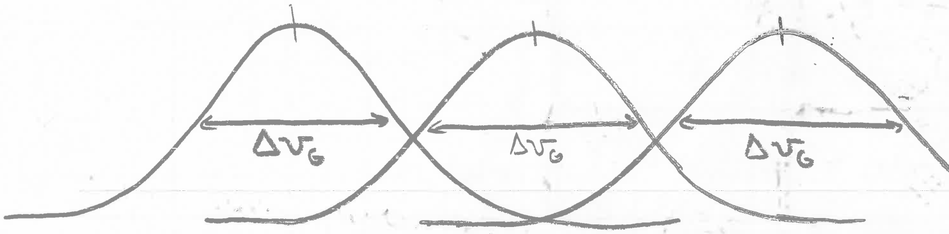
$$\boxed{t_p = 688\ \text{ps}}$$

(+10 points)

2. There are two acceptable ways of doing this problem.

1

① 620 nm ② 700 nm ③ 780 nm



$\tau_G = 40$ fs $\rightarrow$ $\Delta\nu_G \tau_G \geq \frac{1}{2}$

$$\Delta\nu_G \geq \frac{1}{2\tau_G}$$

For each laser, the BW is: $\Delta\nu_G \geq \frac{1}{80\times10^{-15}\,s} = 1.25\times10^{13}$ Hz

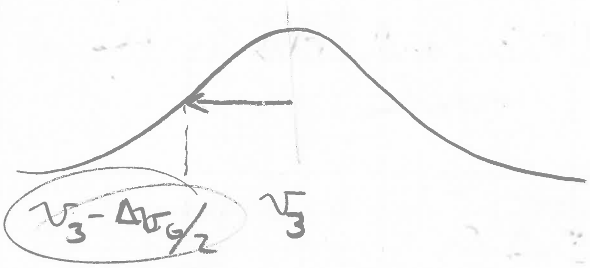
$$\nu_1 = \frac{c}{\lambda_1} = \frac{3\times10^8\ m/s}{620\times10^{-9}\ m} = 4.84\times10^{14}\ Hz$$

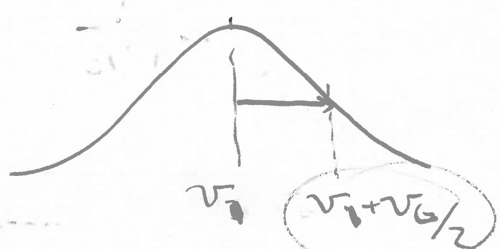
$\Delta = 5.53\times10^{13}$ Hz

$$\nu_2 = \frac{c}{\lambda_2} = \frac{3\times10^8\ m/s}{700\times10^{-9}\ m} = 4.29\times10^{14}\ Hz$$

$\Delta = 4.395\times10^{13}$ Hz

$$\nu_3 = \frac{c}{\lambda_3} = \frac{3\times10^8\ m/s}{780\times10^{-9}\ m} = 3.85\times10^{14}\ Hz$$





3.78×10^{14} Hz

4.90×10^{14} Hz

$$\Delta\nu_G' = \nu_1 + \frac{\Delta\nu_G}{2} - \nu_3 + \Delta\nu_G/2$$

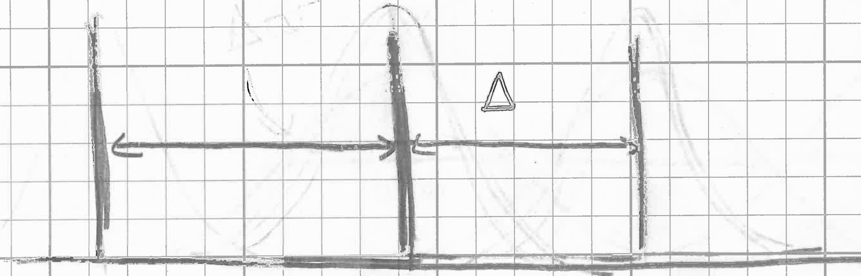
$$= (\nu_1 - \nu_3) + \Delta\nu_G = (4.84 - 3.85)10^{14}\ Hz + 1.25\times10^{13}\ Hz$$

$$= 1.11\times10^{14}\ Hz \rightarrow \tau_G' \geq \frac{1}{2\Delta\nu_G'}$$

In order to do this properly, the field strengths (intensities) of the pulsed beams would need to be precisely adjusted.

$\boxed{\tau_G' \geq 4.5 \text{ fs centered @ } 4.34\times10^{14} \text{ Hz } (691.2 \text{ nm})}$

(2)



Roughly speaking, the freq. spacing between adjacent modes is similar. Approximately, Δ (spacing between lasers) can be equated to the FSR and the laser outputs can be considered longitudinal modes.

Taking Δ @ 700 nm:

$$\Delta = \frac{c}{\lambda^2}\Delta\lambda = \frac{3\times10^{8}\ \text{m/s}}{(7\times10^{-7}\ \text{m})^2}\,80\times10^{-9}\ \text{m}$$

where $\Delta\lambda = 80$ nm

$$\Delta = 4.90\times10^{13}\ \text{Hz}$$

$$\tau_G \simeq \boxed{\frac{1}{2M\Delta}}$$, where $M = 2$ and $M\Delta$ is the total freq. spread

$(\Delta\nu_G'\tau_G' \geq \frac{1}{2}) \nearrow \frac{1}{\Delta\nu_G'}$

$$\tau_G' \simeq \frac{1}{4\times4.9\times10^{13}\ \text{Hz}} = \boxed{5.1\ \text{fs}}$$

Method (2) is not very accurate, but it does give an answer that is similar to the more accurate Method (1).

3. (+15 points)

(a.)

$\lambda = 800$ nm

$W_0 = 25\ \mu m \longrightarrow$ area $= \pi(2.5\times10^{-5}\,m)^2 = 1.96\times10^{-9}\,m^2$

$= 1.96\times10^{-5}\,cm^2$

$\langle n_2\rangle = 3\times10^{-16}\ cm^2/W$

$\tau_0 = 25$ fs $\qquad d = 1$ cm

Rep. rate $= 80\times10^{6}$ Hz

Pulse energy $= 10$ nJ/pulse

$$W_0'^{\,2} = W_0^{\,2}\,\frac{f^2}{f^2+\rho_0^{\,2}}$$

$$\rho_0 = \frac{\pi W_0^2}{\lambda} = \frac{\pi(2.5\times10^{-5}\,m)^2}{8\times10^{-7}\,\cancel{mm}} = 2.5\ mm$$

$$f \simeq f_0\left(1+2\left[t/\tau_0\right]^2\right)$$

$$f_0 = \frac{W_0^2}{4\langle n_2\rangle d I_0} = \frac{(2.5\times10^{-5}\,m)^2}{4\left(3\times10^{-16}\,\frac{cm^2}{W}\right)\left(2.04\times10^{10}\,\frac{W}{cm^2}\right)(10^{-2}\,m)} = 0.00256\,m = \boxed{2.56\ mm}$$

$$I_0 = \text{peak intensity} = \underbrace{\boxed{\frac{10\ nJ}{\cancel{pulse}}\times\frac{\text{pulse}}{1.96\times10^{-5}\,cm^2}}}_{\text{fluence}}\ \frac{1}{25\,fs}$$

$$= 2.04\times10^{10}\ W/cm^2$$

$$W_0' = 25\,\mu m\sqrt{\frac{(2.56\,mm)^2}{(2.56\,mm)^2+(2.5\,mm)^2}} = \boxed{18\ \mu m}$$

$$\rho_0' = \frac{\pi W_0'^{\,2}}{\lambda} = \frac{\pi(1.8\times10^{-5}\,m)^2}{8\times10^{-7}\,m} = \boxed{1.28\ mm}$$

ρ_0' is nearly twice as small as ρ_0

(+ 10 points)

(b.) (i.) Peak power

$$\frac{P_{out}}{P_{in}} \approx \frac{2R^2}{w_0^2 f_0^2}(f_0^2+\rho_0^2)\left[1-\frac{\rho_0^2(2\,t/\tau_0)^2}{f_0^2+\rho_0^2}\right]$$

We want $D = 2R$ for $\frac{P_{out}}{P_{in}} = 0.95$

$$R^2 = \frac{0.95\ w_0^2 f_0^2}{2(f_0^2+\rho_0^2)}\ \frac{1}{\left[1-\frac{\rho_0^2(2t/\tau_0)^2}{f_0^2+\rho_0^2}\right]}$$

$t=0$

$$R = \sqrt{\frac{0.95\,w_0^2 f_0^2}{2(f_0^2+\rho_0^2)}} = 1.54\times10^{-5}\ m$$

$$= 15.4\ \mu m$$

$\boxed{D = 30.8\ \mu m}$

(ii.) Total power

$$I(r,t) = I_0\, e^{-2(t/\tau_0)^2}\, e^{-2r^2/w_0^2}$$

95% of the incident power is transmitted when the aperture is set @ a radius of two std. deviations, where std. deviation $=\sigma$ for a Gaussian of the form $e^{-x^2/2\sigma^2}$

$$\frac{w_0^2}{2} = 2\sigma^2 \longrightarrow \frac{w_0^2}{4} = \sigma^2 \longrightarrow \frac{w_0}{2} = \sigma$$

$$D = 2R = 4\sigma = 2w_0$$

$\boxed{D = 50\ \mu m}$

4. $\tau_6 = 100$ fs

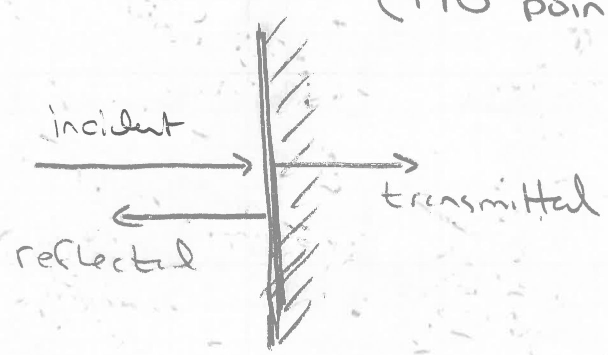
$n = 4$

$d_{spot} = 10\ \mu m = 10^{-3}$ cm

$P = 1$ W

Rep. rate $= 8 \times 10^7$ Hz

(a.) (+10 points)



First, let's calculate the peak E-field in air ($n \cong 1$).

$$\text{Peak intensity} = \boxed{\frac{1\,W}{8\times10^7\,Hz}}\ \boxed{\frac{1}{\frac{\pi}{4}(10^{-3}\,cm)^2}}\ \boxed{\frac{1}{10^{-13}\,sec}}$$

pulse energy area pulse width

$$= 1.59\times10^{11}\ W/cm^2$$

$$\text{Peak E-field} = \sqrt{\frac{2}{cn\epsilon_0} I}\ ,$$

$c = 3\times10^8$ m/s

$n = 1$

$\epsilon_0 = 8.85\times10^{-12}$ F/m

$I = 1.59\times10^{15}$ W/m^2

$$= 1.09\times10^9\ V/m$$

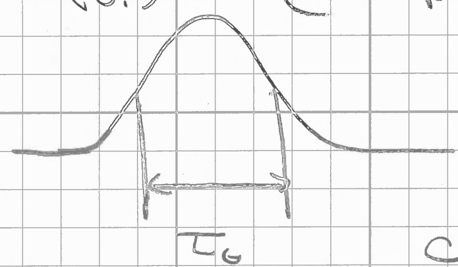
$$= 1.09\times10^7\ V/cm$$

Now, let's include reflection losses:

$$tE_i = E_t\ , \text{ where } t = \frac{2n_1}{n_1+n_2} = \frac{2}{5}$$

Peak E-field inside the semiconductor $= \boxed{4.38\times10^6\ V/cm}$

(b.) (+10 points)



$$\frac{c}{n}\tau_G = \Delta x$$

$$3\times10^{8}\ \mathrm{m/s}\times10^{-13}\ \mathrm{sec} = \Delta x$$

$$3\times10^{-5}\ \mathrm{m} = \Delta x$$

$$\boxed{30\ \mu m \text{ or } 3\times10^{4}\ nm = \Delta x} \quad \text{(before semiconductor)}$$

$$\boxed{7.5\ \mu m \text{ or } 7.5\times10^{3}\ nm = \Delta x} \quad \text{(inside the semiconductor)}$$

(+5 points)

(c.) The pulse in air is 3×10^4 nm long. One optical cycle is 800 nm, so there are

$$\frac{3\times10^{4}\ nm}{800\ nm} = \frac{300}{8} = \boxed{37.5 \text{ optical cycles}}$$