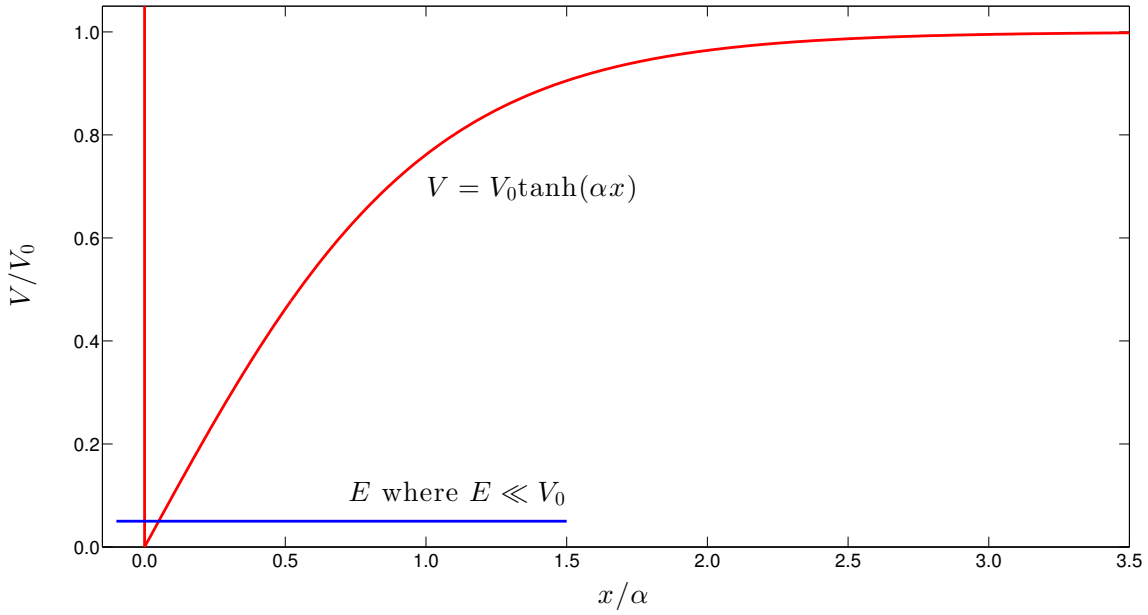
PHYS 4310, Mid-Term 2, Monday November 23rd

1. [16 points] A positively charged trion (one electron and two positive electrons, also known as holes) are in a solid with a dielectric constant $\epsilon = 10\epsilon_0$. In this particular solid, the mass of an electron is $0.1m_e$, while the mass of a hole is m_e.
(a) What is the (estimated) Bohr radius of the trion in nanometers?
(b) What is the binding energy of the trion in terms of the hydrogen binding energy, E_B^{H} (= 13.6 eV)?

2. [20 points] Consider a potential, $V(x)$, such that: $V(x) = \begin{cases} \infty & x \leq 0 \\ V_0 \tanh(\alpha x) & x > 0, \end{cases}$ where $V_0 > 0$ (see figure). Using the WKB approximation, find the bound energy states, E_n. Assume that $E \ll V_0$ and make approximations where appropriate.



3. [10 points] Consider a particle in the infinite three-dimensional well. An operator, $\hat{A}$, is defined such that it only depends on $\hat{p}^2$. What is difference between the expectation value, $\langle A \rangle$, at t and $t + t_0$?

4. [25 points] The three-dimensional Fourier transform of $\psi(r,\theta,\phi)$ to $\Phi(\vec{p})$ for a spherically symmetric ψ is:

$$\Phi(\vec{p}) = \frac{4\pi}{\sqrt{(2\pi\hbar)^3}} \int_0^\infty \psi(r) \left[\frac{\sin(pr/\hbar)}{pr/\hbar}\right] r^2 dr. \tag{1}$$

Find the $\Phi(\vec{p})$ for $\psi_{100}(\vec{r}) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$ (the ground state of a hydrogenic atom with an atomic number Z, where $a = \frac{a_0}{Z}$). Simplify your solution so that it is in the form $C_0 \left[\frac{1}{1+C_1^2}\right]^2$, where C_0 is a constant and C_1 is a function of p.