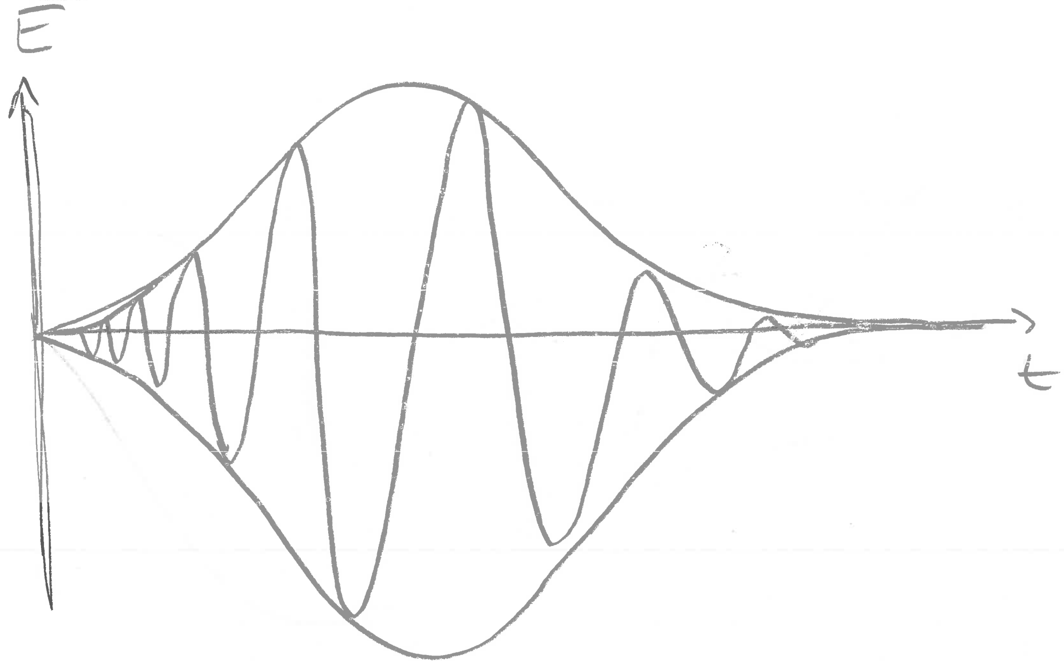
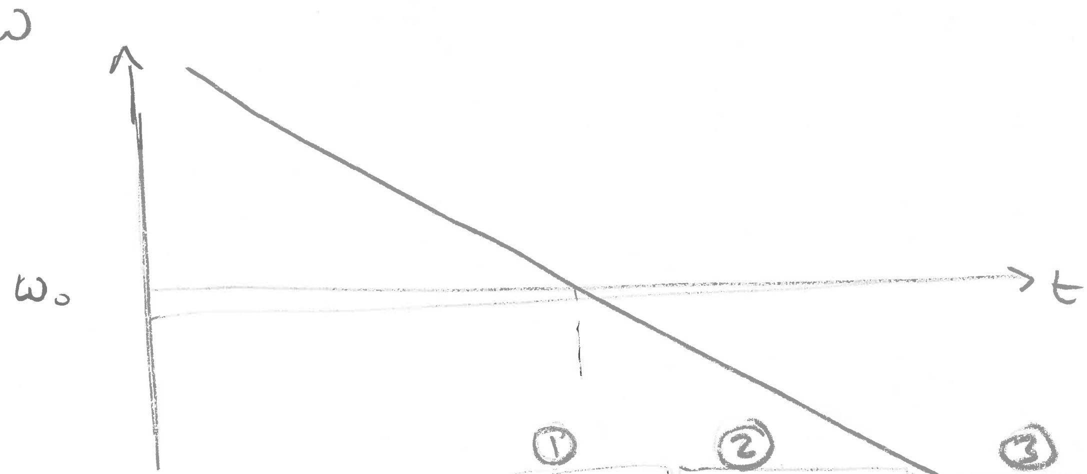
Answers to the Practice Exam

1.

(a.)





(b.)

$$E(\bar{r},t) = \tfrac{1}{2}E_0\, \overset{(1)}{\boxed{e^{-\frac{(x^2+y^2)}{w^2(z)}}}}\ \overset{(2)}{\boxed{e^{i(\omega_0 t - kz)}}}\ \overset{(3)}{\boxed{e^{-(1+ia)(t/\tau_0)^2}}}\ \overset{(4)}{\boxed{e^{i\varphi_0}}}$$

$$\times \underbrace{\left(\cos\omega_0 t\,\hat{x} + \sin\omega_0 t\,\hat{y}\right)}_{\text{circular polarization}} + \text{c.c.}$$

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(1) spatial distribution w/ a beam waist of $w(z)$

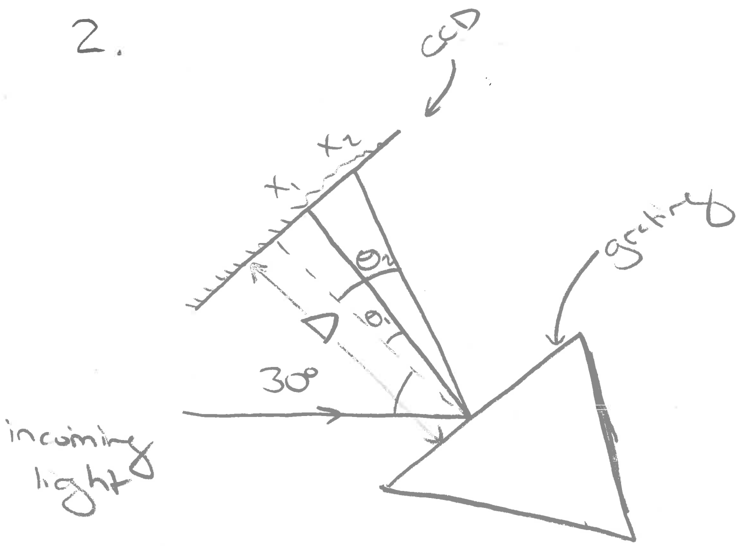
(2) Traveling wave moving @ a velocity of $\frac{\omega_0}{k}$ in the $+\hat{z}$ direction

(3) Temporal pulse description w/ chirp

(4) Carrier-envelop phase

We have eliminated any phase in the polarization, but you could always add a simple α to the arguments of cosine and sine.

2.



$\theta_1 = \measuredangle$ of diffracted $600 - \frac{2\,nm}{2}$ beam

$\theta_2 = \measuredangle$ of diffracted $600 + \frac{2\,nm}{2}$ beam

Grating eqn for 1st-order:

$$\lambda = d(\sin\theta_i + \sin\theta_r)$$

$$\sin\theta_i = \sin 30° = \frac{1}{2}$$

$$d = 10^{-6}\,m = \left(\frac{10^{-3}\,m}{1000\text{ grooves}}\right)$$

$$D = 150\text{ mm}$$

$\therefore \quad \theta_r^i = \sin^{-1}\left(\frac{\lambda^i}{d} - \frac{1}{2}\right)$, where $i = 599$ nm or 601 nm

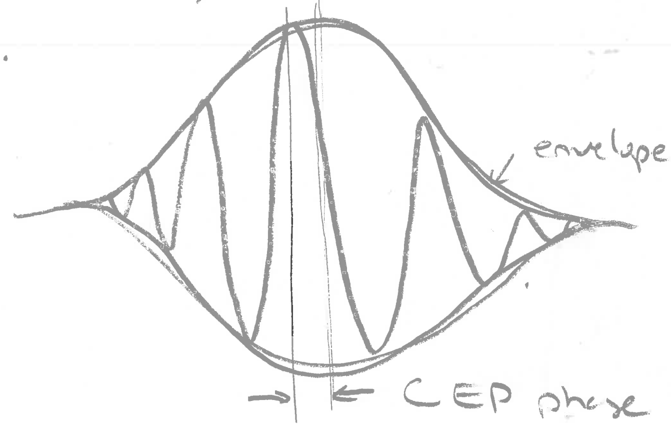
$$x_i = D\tan\theta_r^i \qquad 0.1012 \qquad 0.0992$$

So $\Delta x = x_2 - x_1 = D\left(\tan\theta_r^2 - \tan\theta_r^1\right)$

$= 150\text{ mm}\left[\tan(0.1012) - \tan(0.0992)\right] = 0.2989$ mm

$$\boxed{\Delta x = 298.9\ \mu m \text{ or } 14.9 \text{ pixels}}$$

3. Carrier-envelope phase is the shift (converted to phase) between the pulse envelope relative position and the rapidly changing E-field (carrier) position. CEP develops b/c of a difference between the phase and group velocities. If it changes over time (e.g., pulse to pulse), the electric field will not be consistent during the measurement leading to an "averaged" effect that may prevent phase-sensitive behaviors from being observed.



4. If the thickness of the xtal exceeds l_{coh} (the coherence length), then we are no longer measuring the THz field in an effective manner w/ our NIR probe.

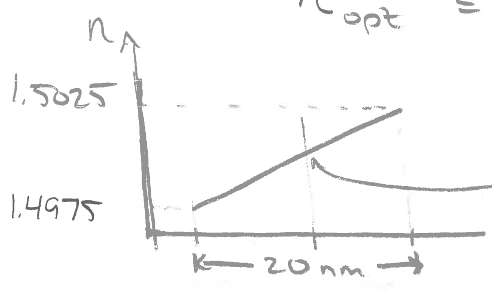
$$l_{coh}(\omega_{THz}) = \frac{\pi c}{2\pi \nu_{THz} \left| n_{opt}^{eff}(\omega_{NIR}) - n_{THz} \right|}$$

$$\nu_{THz} = 1.5 \times 10^{12}\ Hz$$

$$n_{THz} = 1.5$$

$$n_{opt}^{eff} = n_{opt}(\omega_0) - \lambda \left.\frac{dn}{d\lambda}\right|_{\lambda_{opt}} = 1.5 - 800nm \times 2.5 \times 10^{-4}\ nm^{-1}$$

$$= 1.3$$



$$\frac{\Delta n}{\Delta \lambda} \cong \frac{dn}{d\lambda} = \frac{0.0050}{20\ nm} = 2.5 \times 10^{-4}\ nm^{-1}$$

$$l_{coh} = \frac{3\times10^{8}\ m/s}{3\times10^{12}\ Hz\ |1.3-1.5|} = 5\times10^{-4}\ m$$

$$= \boxed{500\ \mu m}$$

The thickness of our xtal should be (slightly) greater than 500 μm.

5.

(a.)

$$|m\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ e^{-i\varphi} \end{bmatrix}$$

$$|m\rangle\langle m| = \frac{1}{2}\begin{bmatrix} 1 \\ e^{-i\varphi} \end{bmatrix}\begin{bmatrix} 1 & e^{i\varphi} \end{bmatrix} = \boxed{\frac{1}{2}\begin{bmatrix} 1 & e^{i\varphi} \\ e^{-i\varphi} & 1 \end{bmatrix}}$$

(b.) For $\varphi = \pi/2$, we have:

$$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix} \rightarrow \begin{vmatrix} \frac{1}{2}-\lambda & i/2 \\ -i/2 & \frac{1}{2}-\lambda \end{vmatrix} = 0$$

$$\left(\frac{1}{2}-\lambda\right)^2 - \frac{1}{4} = 0$$

$$\left(\frac{1}{2}-\lambda\right) = \pm\frac{1}{2}$$

$$\boxed{\lambda = 0 \text{ or } 1}$$

$\underline{\lambda_1 = 0}$

$$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = 0\begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$\alpha + i\beta = 0$$

$$-i\alpha + \beta = 0 \quad \rightarrow \quad i\alpha = \beta$$

$$V_1 = \sqrt{\frac{1}{2}}\begin{bmatrix} 1 \\ i \end{bmatrix} \qquad \text{RCP}$$

$\underline{\lambda_2 = 1}$

$$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$\frac{1}{2}(\alpha + i\beta) = \alpha \quad \rightarrow \quad i\beta = \alpha$$

$$\frac{1}{2}(-i\alpha + \beta) = \beta$$

$$V_2 = \sqrt{\frac{1}{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix} \qquad \text{LCP}$$

The two eigenvectors are RCP and LCP light. If we make a measurement on the density matrix for LCP light, we have a probability of $|\lambda_1|^2 = 0$ of obtaining RCP and $|\lambda_2|^2 = 1$ of obtaining LCP.