PHYS 4310, Final Exam December 14, 2015

1. An electron is in the spin state:

$$\chi = A \begin{bmatrix} 3i \\ -2 \end{bmatrix}. \tag{1}$$

(a). Find A.
(b). Find $\langle S_i \rangle$ for $i = x, y, z$.
(c). What are the "uncertainties" in S_x, S_y, and S_z? Here, "uncertainty" is taken in the same sense as other observables that we have covered. Check to see if these uncertainties, σ_{S_i}, satisfy $\sigma_{S_i}\sigma_{S_j} \leq \frac{\hbar}{2}|\langle S_k \rangle|$.

2. An electron in hydrogen is in the following position and spin state:

$$\Psi\left(\mathbf{r}, s\right) = R_{21}\left(\sqrt{\frac{1}{3}}Y_1^0\chi_+ + \sqrt{\frac{2}{3}}Y_1^1\chi_-\right), \tag{2}$$

where R_{21} and $Y_1^{0,1}$ are defined in the usual way for the hydrogen atom.
(a). You make a measurement of L^2 on this electron. What values might you get and what are the probabilities of each?
(b). You make a measurement of L_z on this electron. What values might you get and what are the probabilities of each?
(c). Same question as (a), except for S^2.
(d). Same questions as (b), except for S_z.
(e). What is the probability density? Hint: $R_{21} = \sqrt{\frac{1}{24}}a_0^{-3/2}\frac{r}{a_0}\exp\left(-\frac{r}{a_0}\right)$, $Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$, $Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\phi}$.
(f). Find $\langle r\rangle$.

3. A particle in a *harmonic oscillator potential* starts out in the following state:

$$\Psi(x,0) = A\left[2\psi_0(x,0) + 3\psi_1(x,0)\right], \tag{3}$$

where ψ_i is the i^{th} harmonic oscillator eigenstate.
(a). Find A.
(b). What is $\Psi(x,t)$ and $|\Psi(x,t)|^2$?
(c). Find $\langle x\rangle$, $\langle x^2\rangle$, $\langle p\rangle$, and $\langle p^2\rangle$.
(d). Check the position-momentum uncertainty relation.

4. The electric dipole operator, $\hat{\mu} = q\hat{\mathbf{r}} = q\hat{x}$ (for 1D), couples transitions from ψ_i to ψ_f if the integral $\int_{-\infty}^{\infty} \psi_i^* \hat{\mu} \psi_f$ is non-zero (here, q is the charge). Put slightly differently, if $\psi_i^* \hat{\mu} \psi_f$ is anti-symmetric, then the transition **cannot** occur (*i.e.*, it is forbidden). Between the four main orbital angular momentum states, s ($l = 0$), p ($l = 1$), d ($l = 2$), and f ($l = 3$), which transitions are **dipole allowed**?

5. The energy splitting of orbital levels for ψ_{nlm} goes as $\Delta E = m\mu_B B$, where μ_B is the Bohr magneton ($\sim$57.88 μeV/T for an electron). Ignoring spin, draw the orbital splitting with and without a magnetic field, B, for the s, p, and d orbital states.

6. In class, we saw that two eigenstates (wavefunctions) are orthogonal when $\int \psi_a^* \psi_b = 0$.

(a). Beyond this formula, what does it mathematically mean when we say that two eigenstates (wavefunctions) are orthogonal?

(b). What does it physically mean when we say that two eigenstates (wavefunctions) are orthogonal?

7. A wavefunction, $\psi(x,0)$, in the infinite square well, of length L, is prepared such that $\psi(x,0) = \frac{1}{\sqrt{2}}[\psi_1 - i\psi_3]$, where ψ_i is the i^{th} infinite square well wavefunction.
(a). Using the explicit form of ψ_i, find $\langle p^2 \rangle$.
(b). What happens to $\langle p^2 \rangle$ if L is tripled and m is halved? What about E_n, where E_n is the energy eigenvalue of the n^{th} level?

8. At what length scales is it applicable to use quantum mechanics? When do we move into the so-called classical regime?