PHYS 4310, Quiz 4 October 27, 2015

1. The pulse intensity from a laser is measured by a spectrometer (Fig. 1). You know that the form of the pulse intensity can closely be approximated by a Gaussian, such that: $I = \exp\left[-\frac{1}{2}\left(\frac{\nu-\nu_0}{\Delta\nu}\right)^2\right]$, where $\nu_0 = 2$ THz and $\Delta\nu$ is the standard deviation of the frequency. Your colleague gives you the e^{-1} points of the pulse (0.283 THz or $0.2\sqrt{2}$ THz), which is a measure of the frequency spread (but not $\Delta\nu$).

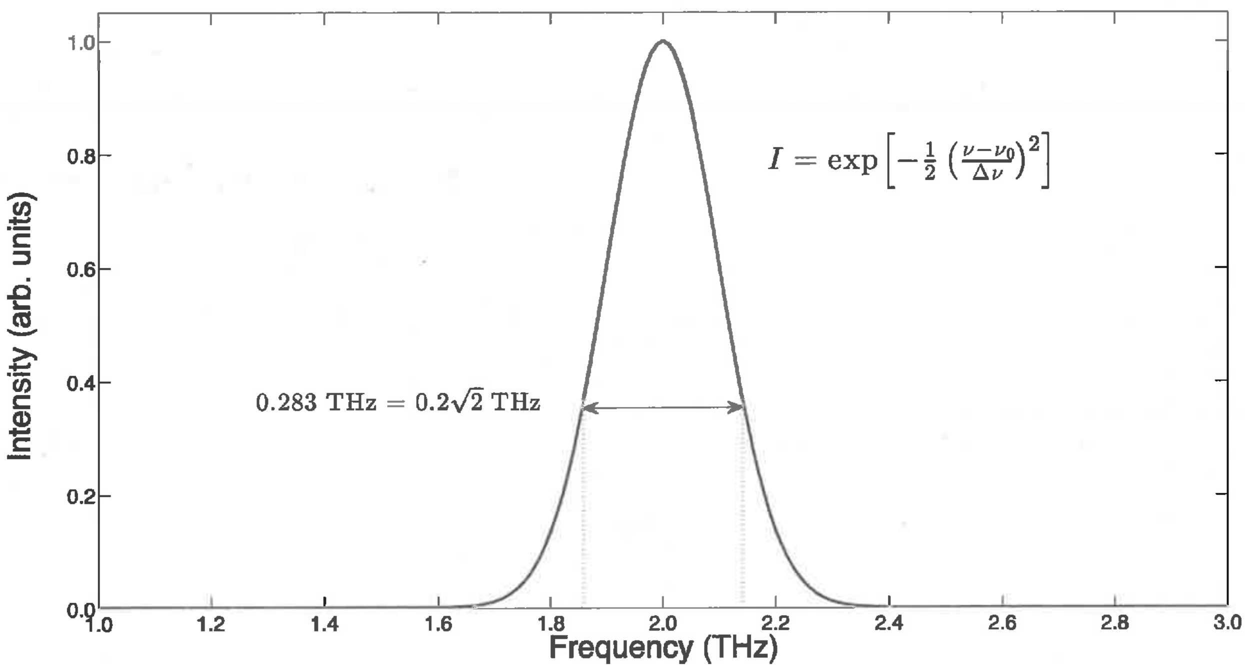

FIG. 1. Frequency components of the measured pulse intensity centered on 2.0 THz. The red dotted lines demarcate the e^{-1} points that your colleague measured for you.

(4 points)

(a.) Using the form of the Gaussian intensity and the fact that these points occur at e^{-1}, find $\Delta\nu$.

$$-1 = -\frac{1}{2}\left(\frac{\nu-\nu_0}{\Delta\nu}\right)^2 \leftarrow \text{condition @ } e^{-1} = I_{\pm\sigma'}$$

$$\pm\sqrt{2}\,\Delta\nu = \nu - \nu_0$$

So if $\nu - \nu_0 = \pm\sqrt{2}\,\Delta\nu$, then the distance between the red lines is $2\sqrt{2}\,\Delta\nu$ or $\Delta\nu = 0.1$ THz.

(4 points)

(b.) The Gaussian waveform is the pulse shape that obtains the minimum uncertainty in ΔE and Δt. Assuming this Gaussian pulse is Fourier-transform limited, use the energy-time uncertainty principle to find Δt.

$$h\Delta\nu = \Delta E$$

$$h\Delta\nu\,\Delta t \geq \hbar/2$$

$$\Delta\nu\,\Delta t \geq \frac{1}{4\pi}$$

$$\Delta t \geq \frac{1}{4\pi\Delta\nu}$$

$\Delta\nu = 0.1\ \text{THz}$ so: $\Delta t \geq \frac{1}{0.4\pi\ \text{THz}} \approx \frac{1}{1.2} 10^{-12}\ \text{sec}$

$$\Delta t \geq \frac{5}{6} 10^{-12}\ \text{sec} \approx 0.8\times 10^{-12}\ \text{sec}$$

$$\boxed{\Delta t \gtrsim 800\ \text{fs}}$$

(2 points)

(c.) A bandpass filter is placed in the beam line substantially reducing the spectral width ($\Delta\nu$) of the pulse. Does your temporal width (Δt) increase or decrease?

If $\Delta\nu$ decreases, Δt increases. The pulse temporally broadens as its spectral content is reduced.