PHYS 5130, Homework Set 2, due at 5 pm on Thursday, Feb. 25$^{\text{th}}$.

1. You are given the normalized absorption of a set of N electric dipole oscillators per unit volume (see Fig. 1).

(a). (15 points) Assuming that oscillating unit is a single electron, what electric field magnitude (in V/m) do you need to drive the dipole a distance of 1 Å? Assuming that you have tuned the laser to exact resonance.

(b). (15 points) Again assuming that you have a monochromatic laser at resonance, how far (in m) will an electric dipole be stretched when 10 W of laser light is incident on the solid over a spot diameter of 100 μm?

(c). (10 points) If $N = 10^{22}$ atoms/cm^3, what is κ and n'?

2. Rabi frequency derivation details:

(a). (10 points) In class, we saw that $e^{-iH't/\hbar} = \cos\left(\frac{\Omega t}{2}\right)\mathbf{I} - i\left(\cos\theta\sigma_z + \sin\theta\sigma_x\right)\sin\left(\frac{\Omega t}{2}\right)$, where $H' = \frac{\hbar\Omega}{2}\left(\cos\theta\sigma_z + \sin\theta\sigma_x\right)$, $\mathbf{I}$ is the identity matrix, and σ_i is the Pauli matrix in the i^{th} direction. Prove this relation.

(b). (20 points) Prove the following: (i) $\sigma_z|\beta\rangle = -|\beta\rangle$ and (ii) $\sigma_x|\beta\rangle = |\alpha\rangle$.

3. A sodium atom is placed in a cavity of volume 1 cm^3 with the walls at a temperature, T, producing a thermal radiation field with a spectral energy density $\rho\left(\nu\right)$.

(a). (15 points) At what temperature does the spontaneous and induced transition probabilities become equal for the $3P \rightarrow 3S$ transition (λ= 589 nm). Hint: the average photon density per unit volume of a thermal radiation field is $\bar{n} = \frac{1}{e^{-h\nu/k_BT}-1}$.

(b). (10 points) For this transition, $\tau\left(3P\right)$= 16 ns. Using the energy-time uncertainty principle and remembering that τ is equal to $\sigma_t/2$, where σ_t is the time it takes an observable to change by one standard deviation, what is the natural frequency of this transition, $\Delta\nu_L$, in MHz?

(c). (10 points) Within the frequency interval, $\Delta\nu_L$, calculate the cavity mode den-

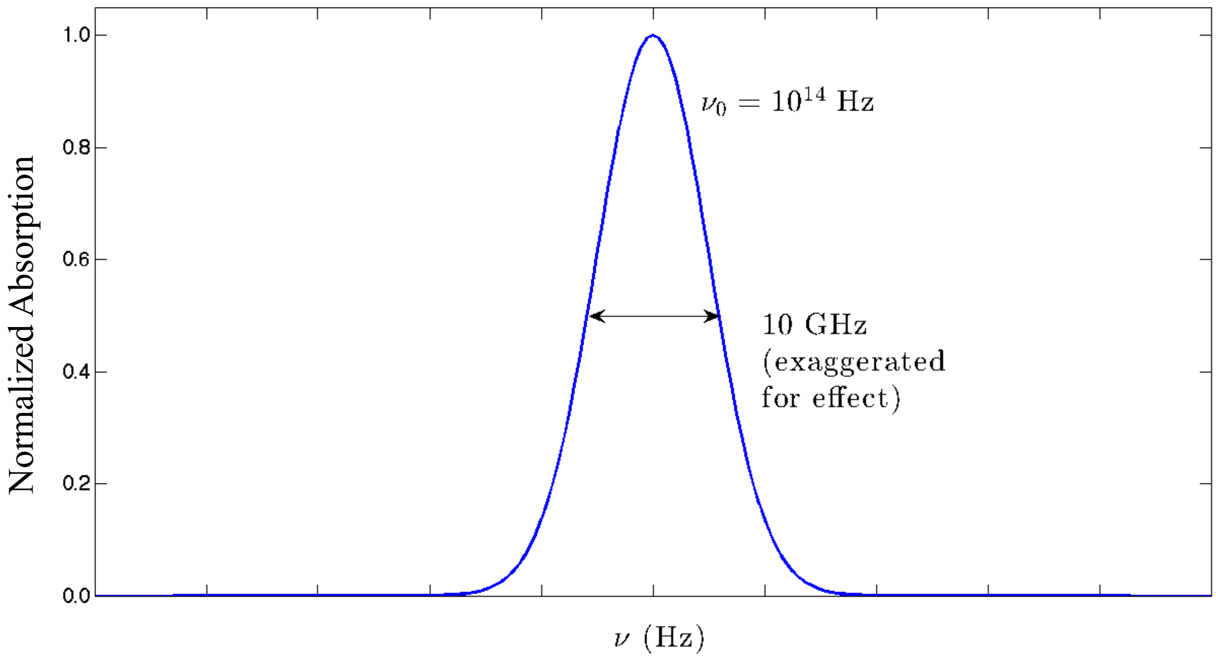

FIG. 1. Normalized absorption centered at 10^{14} Hz. For this curve, the full width at half maximum is 10 GHz.

sity (in cm^{-3}).

(d). (10 points) For one photon per mode for this particular transition at λ=589 nm in this cavity, what is the radiation density in the cavity (in units of W-s/cm^3)?

(e). (15 points) Find the laser intensity, I, that is necessary to achieve this radiation density (assume a frequency spread equal to $\Delta\nu_L$).