# PHYS 4310, Homework 6 due on **Monday** October 19th, 2015

Griffiths 8.2 (18 points), 8.7 (15 points), 8.11 (20 points)

4. (8 points) Proton and deuteron (one proton and one neutron, but no electrons) beams, each with a kinetic energy of 4 MeV are incident on a barrier that is $10^{-12}$ cm thick and 10 MeV high. What is the transmission coefficient of each beam? Partial answer: $T_{\text{deuteron}} = 1.0{\times}10^{-6}$.

5. (15 points) A ball of mass, $m = 0.1$ kg, is bounced off a floor such that its potential energy, $V(x)$, can be defined as: $V(x) = mgh$ for $x > 0$ and $\infty$ for $x \leq 0$, where $g = 9.8$ m/s$^2$ is the acceleration from gravity and $h$ is the height in meters.
**(a).** Solve the Schrödinger equation (*i.e.*, find $\psi$) for this potential in terms of the Airy function, $Ai(\alpha x)$. The $Bi(\alpha x)$ term blows up for large $x$ rendering $\psi$ non-normalizable. There is no need to normalize $\psi$. *Ans:* $\psi(x) = aAi\left[\alpha\left(x - \frac{E}{mg}\right)\right]$.
**(b).** Find the first three allowed energies in joules. *Hint*: If $V(x \leq 0) = \infty$, then what is $\psi(0)$? The zero solutions of $Ai(z) = 0$ are $a_1 = -2.34$, $a_2 = -4.09$, and $a_3 = -5.52$. *Partial answer:* $E_1 = 8.81{\times}10^{-23}$ J.
**(c).** Using the form of $E_n$ we found in part (b), what is the ground state energy ($E_1$) in eV of an electron ($m_e = 9.11 \times 10^{-31}$ kg)? What is $\langle x \rangle$? To find $\langle x \rangle$, you will need to employ the virial theorem: $2\langle T \rangle = \langle x\frac{dV}{dx} \rangle$, where $\langle H \rangle = \langle T \rangle + \langle V \rangle = E_n$.

6. (12 points) **(1)** For what range of $\nu$ is the function $f(x) = x^\nu$ in Hilbert space, on the interval $(0, 1)$? Assume $\nu$ is real (can be positive or negative).
**(2)** For $\nu = 1/2$, is $f(x)$ in this Hilbert space? What about $xf(x)$ and $\left(\frac{d}{dx}f(x)\right)$?

**Helpful tips for 8.11:** Use Eqn. 8.51 to set up your problem, defining your well from 0 to $x_2$, where $x_2$ is the intersection of $E$ and the potential (*i.e.*, what is $E$ in terms of $\alpha$, $x_2$, and $\nu$?). Define $z$ as $\frac{\alpha}{E}x^\nu$ to simplify the integral. $\int_0^1 z^{1/\nu-1}\sqrt{1-z}\,dz = \frac{\Gamma(1/\nu)\Gamma(3/2)}{\Gamma(1/\nu+3/2)}$. From here, you need to play with the $\Gamma$ function to obtain the answer given in the problem (Eq. 8.53). For $\nu$ =2, $E_n$ reduces to $\left(n - \frac{1}{2}\right)\hbar\sqrt{\frac{2\alpha}{m}}$. For the harmonic oscillator, what is $\alpha$? Substitute in this expression for $\alpha$ into $E_n$. How does this compare to the harmonic oscillator?