PHYS 4310, Extra Credit Question (worth one full quiz) due November 30^{th}, 2015.

To convert into momentum space, we can Fourier transform our spatial wavefunction, $\psi(\boldsymbol{r})$ using:

$$\phi(\boldsymbol{p}) \equiv \frac{1}{(2\pi\hbar)^{3/2}} \int e^{-i(\boldsymbol{p}\cdot\boldsymbol{r})/\hbar} \psi(\boldsymbol{r})\, d^3\boldsymbol{r}. \tag{1}$$

(a). (20 points) Using spherical coordinates and setting the polar axis along $\mathbf{p}$, find $\phi(\boldsymbol{p})$ for the ground state of hydrogen. Do the θ integral first.

(b). (20 points) Show that $\phi(\boldsymbol{p})$ is normalized. Using a computer, plot (separately) $\psi(\boldsymbol{r})$ versus $\mathbf{r}$ and $\phi(\boldsymbol{p})$ versus $\mathbf{p}$.

(c). (20 points) Calculate $\langle r^2 \rangle$ (in real space) and $\langle p^2 \rangle$ (in momentum space) for the ground state of hydrogen.

(d). (10 points) What is the expectation value of the kinetic energy in the ground state? Express your answer as a multiple of E_1 (-13.6 eV; the Rydberg constant).

(e). (15 points) The three-dimensional virial theorem (for stationary states) can be stated as: $2\langle T \rangle = \langle \boldsymbol{r} \cdot \nabla V \rangle$. Using this relation, show that $\langle T \rangle = -E_n$ and $\langle V \rangle = 2E_n$.

(f). (5 points) Check that your answer in part (d) is consistent with the relation that you proved in part (e).