PHYS 4310, Mid-Term 1

1. A free electron (mass = m_e) with an energy of 40 MeV is produced in a lab. Compared to a free proton (mass $\approx 1800 m_e$) with an energy of 200 keV, what is the ratio of the electron to the proton wavelength (*i.e.* what is $\frac{\lambda_e}{\lambda_p}$)?

2. A wavefunction in the ground state of an infinite well of length L, where $V(x) = 0$ on $x\,[0, L]$ and $= \infty$ otherwise, is prepared: $\psi_1 = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L}x\right)$. Suddenly, the well width is expanded from L to $3L$. What is the **probability** of measuring the new system and finding the particle in the $n = 3$ eigenstate of the new well? Hint: $\sin^2\theta = \frac{1-\cos 2\theta}{2}$.

3. A particle of mass, m, is found to be described by a wavefunction, Ψ, defined as:

$$\Psi\left(x,t\right)=\begin{cases} Ax^{1/2}e^{-(x/L)^2}e^{ik_0x}e^{-i\omega_0 t} & x\geq 0 \\ 0 & x<0. \end{cases}$$

(a). Obtain the probability density. Sketch it as a function of x.
(b). Normalize $\Psi\left(x,t\right)$ to find A.
(c). Use your answer in (a) to find help you find $\langle x\rangle$. Being careful of the integration bounds, you may recall that $\int_{-\infty}^{\infty} z^2 e^{-\lambda z^2} dz = \frac{1}{2\lambda}\sqrt{\frac{\pi}{\lambda}}$.

4. Consider a particle of mass m moving in a 1D attractive potential, such that the ground state wavefunction is $\psi_0(x) = \sqrt{\beta} e^{-\beta|x-a|}$, where $\beta > 0$ and a is real and positive. At $t = 0$, the wavefunction of our particle is: $\psi(x, 0) = \psi_0(x) e^{ik_0(x-a)}$.

(a). Find the wavefunction in the momentum basis, $\phi(k) = \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} \psi(x) e^{-ikx} dx$. Hint: after setting up the integral, define $z \equiv x - a$, pull out the e^{-ika} constant, and then split up your integral to handle the absolute value sign.

(b). Find $|\phi(k)|^2$ (momentum probability density) and sketch this function versus k. From this sketch (no calculations allowed!), what is the expectation value of momentum, $\langle p \rangle = \hbar \langle k \rangle$?