1. $p_e = \sqrt{2m_eE_e} \longrightarrow \lambda_e = h/p_e$

$p_p = \sqrt{2m_pE_p} \longrightarrow \lambda_p = h/p_p$

$$\frac{\lambda_e}{\lambda_p} = \frac{\cancel{h}/p_e}{\cancel{h}/p_p} = \frac{p_p}{p_e} = \sqrt{\frac{m_pE_p}{m_eE_e}} = \sqrt{\frac{\overset{9}{\cancel{1800\cdot 200}}\text{ keV}}{\underset{2}{\cancel{400000}}\text{ keV}}} = 3$$

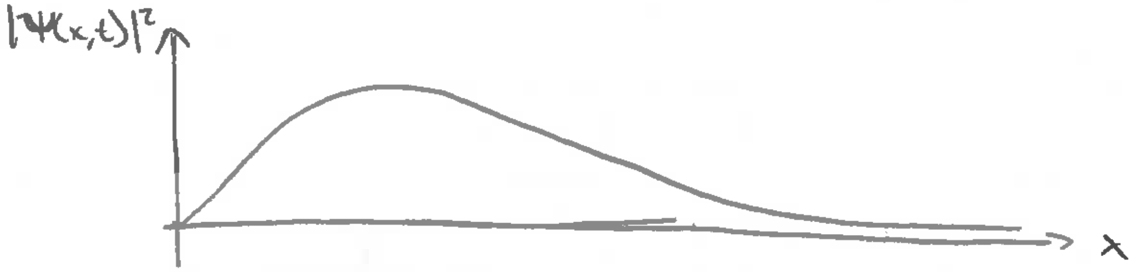
2. $$|\langle\psi_3'|\psi_1\rangle|^2 = |c_{31}|^2 = \left[\sqrt{\frac{2}{L}}\sqrt{\frac{2}{3L}}\int_0^L \sin\left(\frac{\pi}{L}x\right)\sin\left(\frac{\pi}{L}x\right)dx\right]^2$$

$$= \left[\frac{2}{L}\sqrt{\frac{1}{3}}\int_0^L\left[\frac{1-\cos 2\left(\frac{\pi}{L}x\right)}{2}\right]dx\right]^2 = \frac{1}{3}$$

3. $$\psi(x,t) = \begin{cases} Ax^{1/2}e^{-(x/L)^2}e^{ik_0x}e^{-i\omega_0t} & x\geq 0 \\ 0 & x<0 \end{cases}$$

(a.) $|\psi(x,t)|^2 = A^2xe^{-2(x/L)^2}$ for $x\geq 0$

$= 0$ $\quad x<0$



(b.) $1 = A^2\int_0^\infty xe^{-2(x/L)^2}dx \qquad z = \frac{2x^2}{L^2} \qquad dz = \frac{4x}{L^2}dx$

$1 = A^2\int_0^\infty \frac{L^2}{4}e^{-z}dz \longrightarrow \boxed{A = \frac{2}{L}}$

(c.) $\langle x\rangle = \frac{4}{L^2}\int_0^\infty x^2e^{-2(x/L)^2}dx \Rightarrow \langle x\rangle = \frac{4}{L^2}\int_0^\infty x^2e^{-\frac{2}{L^2}x^2}dx = \frac{\cancel{4}}{L^2}\frac{1}{\cancel{4}\left(\frac{2}{L^2}\right)}\sqrt{\frac{\pi}{2/L^2}}$

$\boxed{\langle x\rangle = \frac{L}{2}\sqrt{\frac{\pi}{2}}}$

4. $$\Psi(x,0) = \sqrt{\beta}\, e^{-\beta|x-a|}\, e^{ik_0(x-a)}$$

(a.) $$\phi(k) = \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} \sqrt{\beta}\, e^{-\beta|x-a|}\, e^{ik_0(x-a)}\, e^{-ikx}\, dx$$

$$z \equiv x-a \qquad dz = dx$$

$$\phi(k) = \sqrt{\frac{\beta}{2\pi}} \int_{-\infty}^{\infty} e^{-\beta|z|}\, e^{ik_0 z}\, e^{-ik(z+a)}\, dz$$

$$= e^{-ika}\sqrt{\frac{\beta}{2\pi}}\left[\int_0^{\infty} e^{-\beta z}\, e^{i(k_0-k)z}\, dz + \int_0^{\infty} e^{-\beta z}\, e^{-i(k_0-k)z}\, dz\right]$$

$$\eta = \beta - i(k_0-k) \qquad \gamma = \beta + i(k_0-k)$$

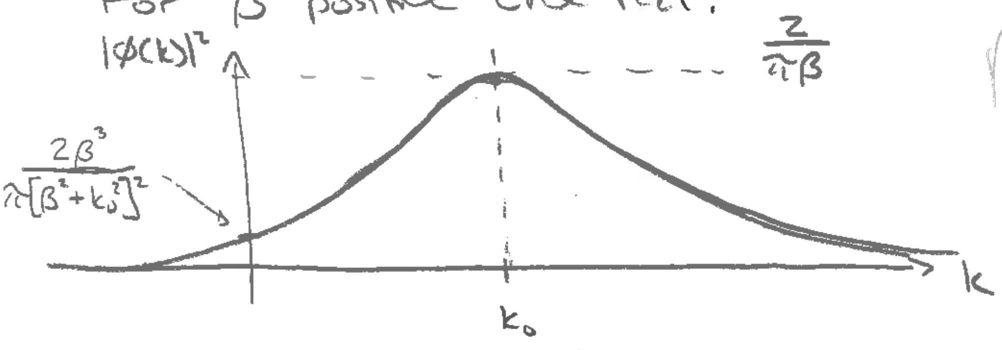
$$= e^{-ika}\sqrt{\frac{\beta}{2\pi}}\left[\frac{-1}{\eta} e^{-\eta z}\Big|_0^{\infty} + \frac{-1}{\gamma} e^{-\gamma z}\Big|_0^{\infty}\right]$$

$$= e^{-ika}\sqrt{\frac{\beta}{2\pi}}\left[\frac{1}{\eta} + \frac{1}{\gamma}\right] = e^{-ika}\sqrt{\frac{\beta}{2\pi}}\left[\frac{1}{\beta - i(k_0-k)} + \frac{1}{\beta + i(k_0-k)}\right]$$

$$= e^{-ika}\sqrt{\frac{\beta}{2\pi}}\left[\frac{\beta + i(k_0-k) + \beta - i(k_0-k)}{\beta^2 + (k_0-k)^2}\right] = \boxed{e^{-ika}\sqrt{\frac{\beta}{2\pi}}\left[\frac{2\beta}{\beta^2+(k_0-k)^2}\right]}$$

(b.) $$|\phi(k)|^2 = \frac{\beta}{\cancel{2}\pi}\left[\frac{\cancel{4}2\beta^2}{[\beta^2+(k_0-k)^2]^2}\right] = \boxed{\frac{2\beta^3}{\pi[\beta^2+(k_0-k)^2]^2}}$$

For β positive and real:



$$\boxed{\hbar\langle k\rangle = \langle p\rangle = \hbar k_0}$$