PHYS 1220 Exam 1, 150 points
September 29th, 2016

$\bar{x} = 70.73$ out of 138
$\sigma_x = 20.28$

1. **Finding the electric field, E, using Gauss's law. (30 points)**

The charge density of a sphere, ρ, is: $\rho(r \le r_0) = Ar^2$, where A is a constant and r_0 is the sphere radius.

(a). (10 points) As a function of r, find $E(r)$ for $r \le r_0$.

$$\oiint \bar{E} \cdot d\bar{a} = \frac{1}{\epsilon_0}\int_V \rho \, dV$$

$$|E| 4\pi r^2 = \frac{1}{\epsilon_0}\int_0^r\int_0^\pi\int_0^{2\pi} A r^2 r^2 \sin\theta \, dr \, d\theta \, d\varphi$$

$$|E| 4\pi r^2 = \frac{1}{\epsilon_0} 4\pi A \frac{r^5}{5}$$

$$\boxed{\bar{E}(r) = \frac{Ar^3}{5\epsilon_0}\hat{r} \text{ for } r \le r_0 \quad \text{or} \quad |E(r)| = \frac{Ar^3}{5\epsilon_0} \text{ for } r \le r_0}$$

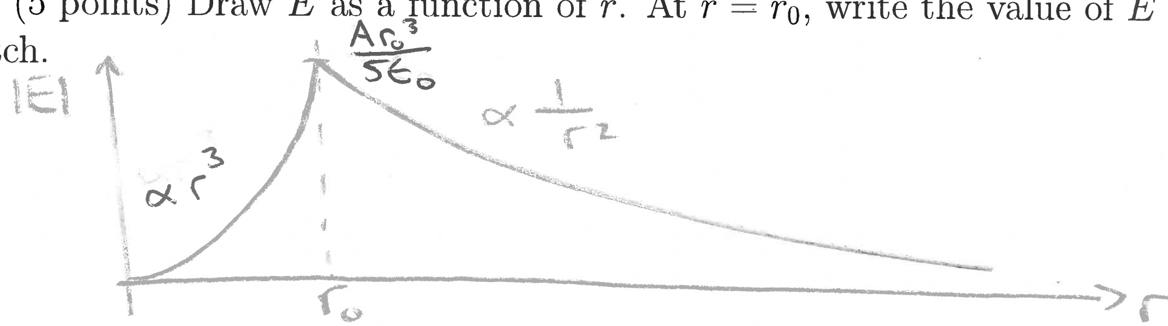
(b). (5 points) As a function of r, find $E(r)$ for $r \ge r_0$.

$$\oiint E \cdot da = \frac{1}{\epsilon_0}\int_V \rho \, dV \qquad |E| = \frac{Ar_0^5}{5\epsilon_0 r^2}$$

$$|E| 4\pi r^2 = \frac{1}{\epsilon_0}\int_0^{r_0}\int_0^\pi\int_0^{2\pi} A r^2 r^2 \sin\theta \, dr \, d\theta \, d\varphi \qquad \boxed{\bar{E}(r) = \frac{Ar_0^5}{5\epsilon_0 r^2}\hat{r}}$$

$$|E| 4\pi r^2 = \frac{1}{\epsilon_0} A \frac{r_0^5}{5} 4\pi$$

(c). (5 points) Draw E as a function of r. At $r = r_0$, write the value of E on your sketch.



(d). (10 points) What is the electric potential difference between $r = 0$ and $r = r_0$ [*i.e.*, what is $V(0) - V(r_0)$]?

$$V = -\frac{Ar^4}{20\epsilon_0}$$

$$V(0) - V(r_0) = \frac{Ar_0^4}{20\epsilon_0}$$

$$V_a - V_a = \int_a^b \bar{E} \cdot d\bar{l} = \int_0^{r_0} \frac{Ar^3}{5\epsilon_0}\hat{r} \cdot dr\hat{r} = \frac{Ar_0^4}{20\epsilon_0}$$

2. Electric dipole in a field. (20 points)
We have two, 1 μC charges of equal, but opposite, value at $(0, -2\text{ m})$ (positive charge) and $(0, +2\text{ m})$ (negative charge).

(a). (4 points) What is $\mathbf{p}$, the dipole moment (remember, $\mathbf{p}$ is a vector)?

$$\bar{p} = q\bar{d} \Rightarrow \bar{p} = 10^{-6}\,\text{C} \times 4\,\text{m}\,(-\hat{y}) = \boxed{-4\times10^{-6}\ \text{C-m}\ \hat{y}}$$

An external electric field, $\mathbf{E}_{\text{ext}} = E_0 \times \frac{1}{2}\left(\hat{x} - \sqrt{3}\hat{y}\right)$, is applied to the dipole. Here, E_0 is 10 N/C.
(b). (4 points) What is the potential energy, U? (Remember: $U = -\mathbf{p}\cdot\mathbf{E}$.)

$$U = -\bar{p}\cdot\bar{E} = -\left[-4\times10^{-6}\ \text{C-m}\ \hat{y}\right]\cdot\left[5\times\text{N/C}\,(\hat{x} - \sqrt{3}\hat{y})\right]$$
$$= \boxed{-3.5\times10^{-5}\ \text{J}}$$

(c.) (7 points) The dipole rotates from its current orientation to its lowest-energy orientation. How much work is done on the dipole by the field?

$$W_{a\to b} = U_a - U_b$$
$$U_a = -3.5\times10^{-5}\ \text{J}$$
$$U_b = -pE_0 = -4\times10^{-6}\ \text{C-m}\times10\ \text{N/C}$$
$$W_{a\to b} = -3.5\times10^{-5}\ \text{J} - (-4\times10^{-5}\ \text{J}) = \boxed{5.0\times10^{-6}\ \text{J}}$$

(d). (5 points) What is the torque, τ, of this dipole? (Remember: $\tau = \mathbf{p}\times\mathbf{E}$.)

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & -4\times10^{-6}\ \text{C-m} & 0 \\ 5\ \text{N/C} & -5\sqrt{3}\ \text{N/C} & 0 \end{vmatrix} = -(-20\times10^{-6}\ \text{N-m}\ \hat{z})$$
$$= \boxed{2.0\times10^{-5}\ \text{N-m}\ \hat{z}}$$

3. Millikan Oil-Drop Experiment. (20 points)

In 1909, Robert A. Millikan and Harvey Fletcher performed an experiment to measure the charge of the electron. Using charged drops of oil, they tuned the voltage between two plates, so that an electrical force, $\vec{F}_{electrical}$, was applied equally and oppositely to the gravitational force, $\vec{F}_{gravitational}$ (see Fig. 1).

(a). (13 points) If the density of oil is 0.9 g/cm^3 and the radius of the oil drop is $r = 1\ \mu$m, what is the field necessary to perfectly suspend the oil drop. Assume it has 10 net electrons on it?

$$m = 0.9\ \text{g/cm}^3 \times \frac{4}{3}(10^{-4}\text{cm})^3 = 1.2\times10^{-12}\ \text{g} = 1.2\times10^{-15}\ \text{kg}$$

$$\bar{F}_g = m\bar{g} = 1.2\times10^{-15}\ \text{kg} \times 9.8\ \text{m/s}^2(-\hat{y}) = -1.176\times10^{-14}\ \text{N}\ \hat{y}$$

$$\bar{F}_e = q\bar{E} = (10e)(-1.6\times10^{-19}\ \text{C/e})\bar{E} = -1.6\times10^{-18}\ \text{C}\ \bar{E}$$

$$|\bar{F}_e| = |\bar{F}_g| \Rightarrow |\bar{E}| = \frac{1.176\times10^{-14}\ \text{N}}{1.6\times10^{-18}\ \text{C}} = \boxed{7.4\times10^{3}\ \text{N/C} \quad \text{or} \quad \bar{E} = 7.4\times10^{3}\ \text{N/C}\ \hat{y}}$$

(b). (7 points) Let's say that the uncertainty of the oil-drop mass is 1% ($= \frac{\delta m}{m}$), the uncertainty in gravity is 0.75% ($= \frac{\delta g}{g}$), and the uncertainty in our applied field is 1.5% ($= \frac{\delta E}{E}$). What is the uncertainty in q ($= \frac{\delta q}{q}$), the charge of the electron?

$$q = \frac{mg}{E} \Rightarrow \frac{\delta q}{q} = \sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta g}{g}\right)^2 + \left(\frac{\delta E}{E}\right)^2}$$

$$= \sqrt{(0.01)^2 + (0.0075)^2 + (0.015)^2}$$

$$\frac{\delta q}{q} = 0.0195 \longrightarrow \boxed{\frac{\delta q}{q} = 2.0\%}$$

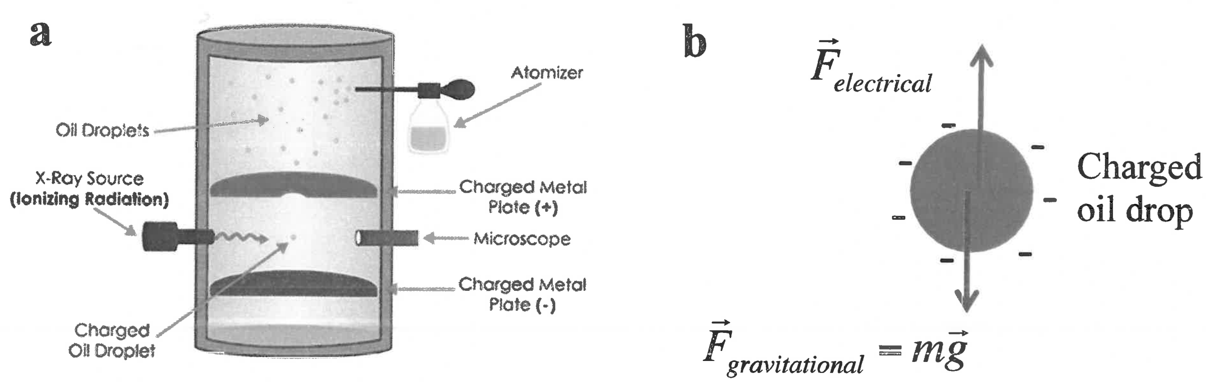


FIG. 1. **a.** Schematic of Millikan's oil-drop experiment to determine elementary charge. An charged oil drop was suspended between two charged plates (figure from www.learner.org/courses/chemistry/images/text_img/millikan_oil_drop.jpg). **b.** Illustration of the forces exerted on the oil drop. When the oil drop is perfectly suspended, $\vec{F}_{electrical} = -\vec{F}_{gravitational}$.

4. Discharging Capacitor. (15 points)

We are given data from a discharging capacitor circuit, which follows the time-dependent voltage form: $V(t) = V_0 * e^{-t/RC}$, where V_0 is the initial voltage at $t = 0$ seconds, t is time, R is the resistor value, and C is the capacitor value ($C = 10^{-9}$ Farad). The data is plotted in Fig. 2 with a best-fit slope of -527 s^{-1} and intercept of 1.27.

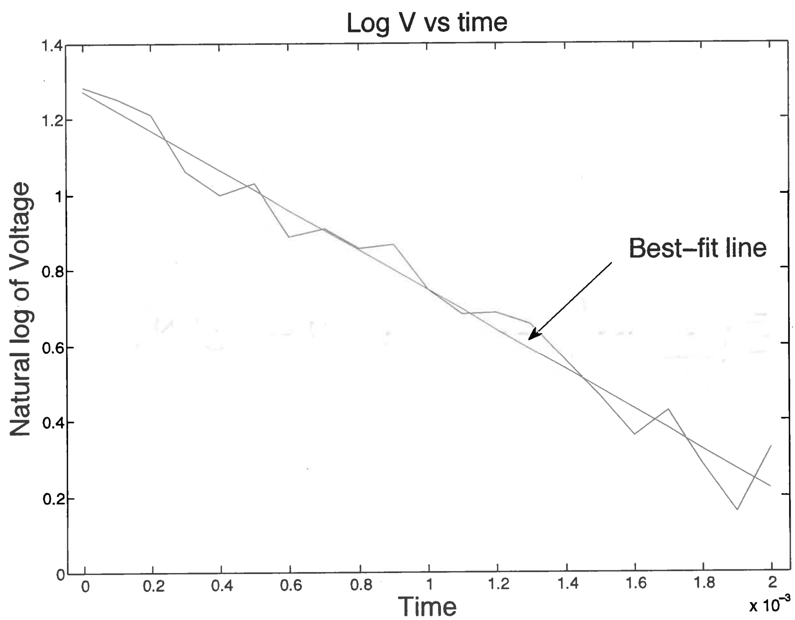


FIG. 2. Natural logarithm of time-dependent data acquired from a discharging capacitor (blue) with a best-fit line (red).

(a). (6 points) What is R in ohms?

$\ln V = \ln V_0 - t/RC$

$527\,s^{-1} = \frac{1}{RC}$

$R = \frac{1}{527\,s^{-1} \times 10^{-9}\,F} = \boxed{1.90\ M\Omega}$

(b). (5 points) What is V_0 in volts?

$\ln V_0 = 1.27$

$\boxed{V_0 = 3.56\ V}$

(c). (4 points) What are two errors that you can find in the format and presentation of this graph?

① Informal title

② No legend

③ No units on x-axis

④ No points for actual data

5. **Forces from an electrostatic ensemble.** (~~30~~ 18 **points)**
There are four charges arranged in a square pattern (see Fig. 3).

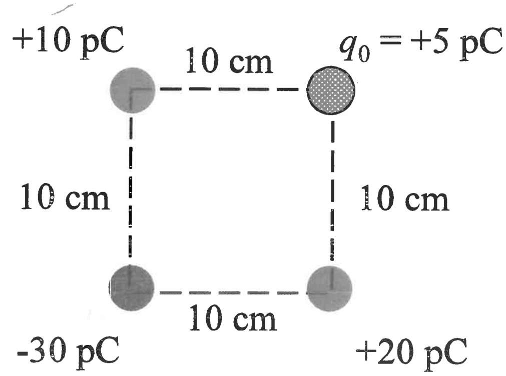


FIG. 3. Magnitude and relative locations of four charges on a square.

(a). (10 points) Find the force on q_0. Remember: force is a vectorial quantity.

$$F_x = K\left(\frac{10^{-11}C \times 5\times10^{-12}}{(0.1m)^2}(+\hat{x}) + \frac{-3.0\times10^{-11}C\times5\times10^{-12}C}{(0.1\times\sqrt{2}\ m)^2}\cos 45^\circ\ \hat{x}\right)$$

$$= -2.7\ pN\ \hat{x}$$

$$\boxed{\bar{F} = -2.7 pN\hat{x} + 42 pN\hat{y}}$$

$$F_y = K\left(\frac{2\times10^{-11}C\times5\times10^{-12}C}{(0.1m)^2}\hat{y} - \frac{3.0\times10^{-11}C\times5\times10^{-12}C}{(0.1\sqrt{2}\ m)^2}\sin 45^\circ\ \hat{y}\right) = 42\ pN\ \hat{y}$$

(b). (8 points) What is the total potential energy of this four-charge configuration.

$$U = K\sum_{i<j}\frac{q_i q_j}{r_{ij}} = K\left[\frac{1\times10^{-11}C\times5\times10^{-12}}{0.1m} + \frac{-3\times10^{-11}\times5\times10^{-12}}{(0.1\cdot\sqrt{2}\ m)} + \frac{2\times10^{-11}C\times5\times10^{-12}C}{0.1m}\right.$$

$$\left. - \frac{1.0\times10^{-11}C\times3\times10^{-11}C}{0.1m} + \frac{1\times10^{-11}C\times2\times10^{-11}C}{0.1\times\sqrt{2}\ m} - \frac{3\times10^{-11}C\times2\times10^{-11}C}{0.1m}\right]$$

$$\boxed{U = -64\ pJ}$$

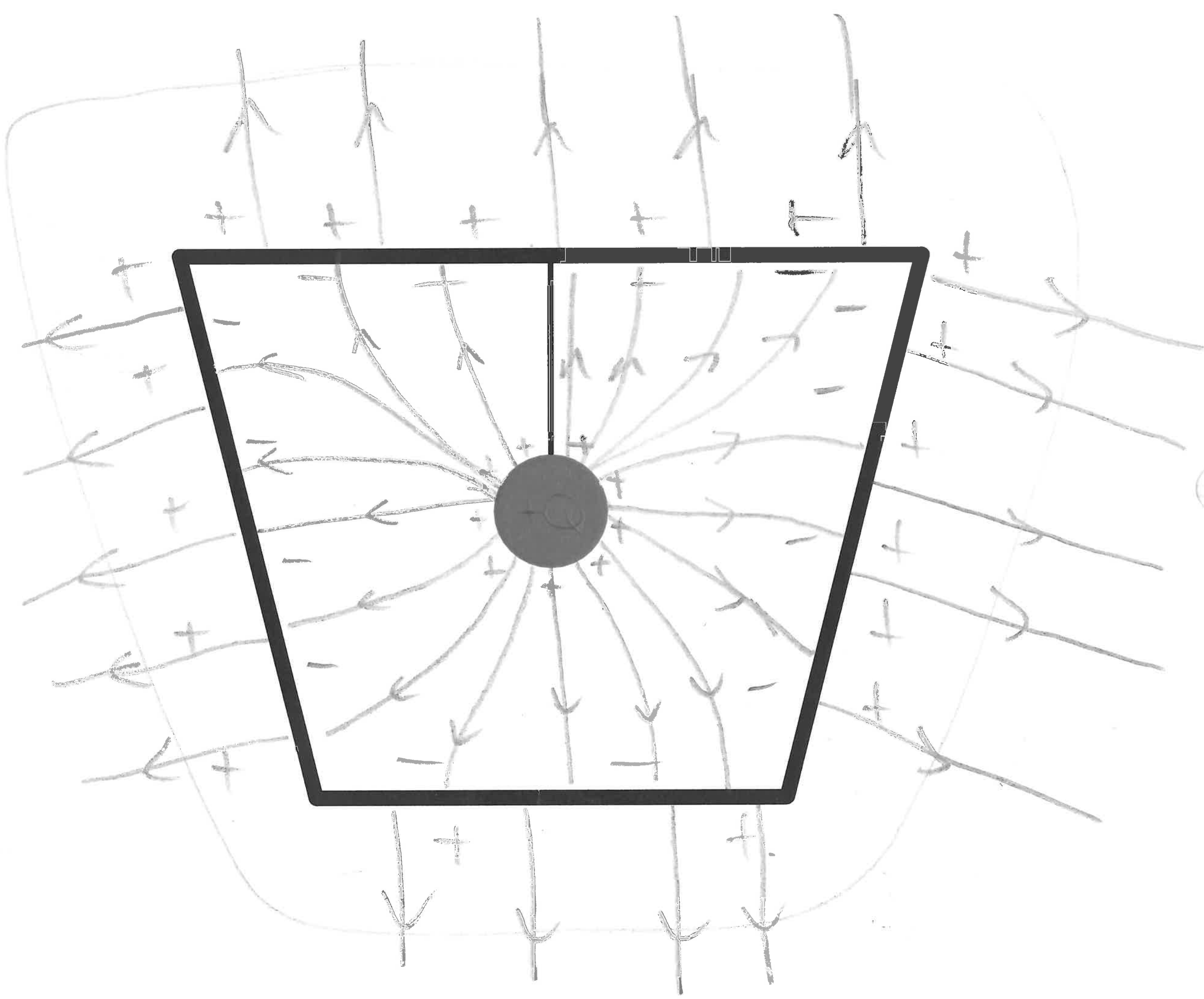
~~(c). (12 points) What is the *external force* needed to bring q_0 from ∞ to its current location?~~

Consider the first three terms above:

$U_{q_0} = 3.954$ pJ $= W_{\infty \to r}$ (work done by the field of the charges)

W done by the external field is therefore = -3.954 pJ. $= -4.0$ pJ (significant figures)

6. **(Partial) Faraday Ice-Pail Experiment. (10 points)**
You have a hollow metal container that you suspend a positive charge, Q, inside of using a charged pith ball attached to a string hanging from the container lid.
(a). (4 points) Draw the charge distribution on the figure below.
(b). (4 points) Draw the electric field lines on the figure below.
(c). (2 points) Draw one line equipotential line outside the container.



Multiple Choice Questions. (25 points)

7. (5 points) Two long conducting cylindrical shells are coaxial and have radii of 10 mm and 500 mm. The electric potential of the inner conductor, with respect to the outer conductor, is +1000 V. An electron is released from **rest** at the surface of the outer conductor. What is the speed of the electron as it reaches the inner conductor? (e = 1.60*10^{-19} C, m_e=9.11*10^{-31} kg). Remember: (1) kinetic energy is equal to $\frac{1}{2}mv^2$, where m is mass and v is velocity, and (2) change in kinetic energy is equal to $-\Delta U$.

(A) 3.8$*$10^4 m/s (B) 1.9$*$10^7 m/s (C) 3.8$*$10^7 m/s (D) 1.9$*$10^6 m/s

8. (5 points) We are given a force of the form: $\vec{F} = xy^2z\hat{\imath} + x^2y^3\hat{\jmath} - 3z^2\hat{k}$. Remembering that if $\vec{\nabla} \times \vec{F} = 0$, the force is conservative, what is $\vec{\nabla} \times \vec{F}$?

(A) 0 (B) $y^2\hat{\imath} + 3x^2y^2\hat{\jmath} - 6z\hat{k}$ (C) $\hat{\jmath} + \hat{k}$ (D) $xy^2\hat{\jmath} + (2xy^3 - 2xyz)\,\hat{k}$

9. (7 points) A huge (essentially infinite) horizontal (i.e., occupying the $x-y$ plane) non-conducting sheet 20.0 cm thick has charge uniformly spread over both faces. The upper face carries -100.0 pC/m^2 while the lower face carries -10.0 pC/m^2. ϵ_0 = 8.85$*$10^{-12} C^2/N-m^2. Circle both the direction and the electric field magnitude at a point within the sheet 1.00 cm below the upper face:

4 points (A) 5.1 N/C (B) 0.0 N/C (C) 90 N/C (D) 10.2 N/C (E) 500 N/C

(A) $\hat{x}$ (B) $-\hat{x}$ (C) $\hat{y}$ (D) $-\hat{y}$ (E) $\hat{z}$ (F) $-\hat{z}$

3 points

10. (6 points) The electric potential in a region is given by (in volts): $V(x, y, z) = 3x^{-2} + 4yz^3 + z^2$. A charge, $-2e$, is moved by the field from $(1, 0, 0)$ to $(1, 1, 1)$. How much work is done by the field on this charge?

(A) 5 V (B) -10 V (C) 10 V (D) $-10e$ J (E) $5e$ J $\boxed{\text{(F) } 10e \text{ J}}$

$-2e(V_a(1,0,0) - V_b(1,1,1)) = (3 - [3+4+1])(-2e)$

$= 10\,e\,J$

11. (2 points) What things in class do you enjoy? What things in class do you want to see changed?