2.10 (a.) $\psi_n = \frac{1}{\sqrt{n!}}(\hat{a}_+)^n \psi_0$ (Eqn. 2.67)

$$\psi_0 = \sqrt[4]{\frac{m\omega}{\pi\hbar}}\, e^{-\frac{m\omega}{2\hbar}x^2} \qquad [\text{Eqn. } 2.59]$$

$$\psi_1 = \sqrt[4]{\frac{m\omega}{\pi\hbar}}\sqrt{\frac{2m\omega}{\hbar}}\, x\, e^{-\frac{m\omega}{2\hbar}x^2} \qquad [\text{Eqn. } 2.62]$$

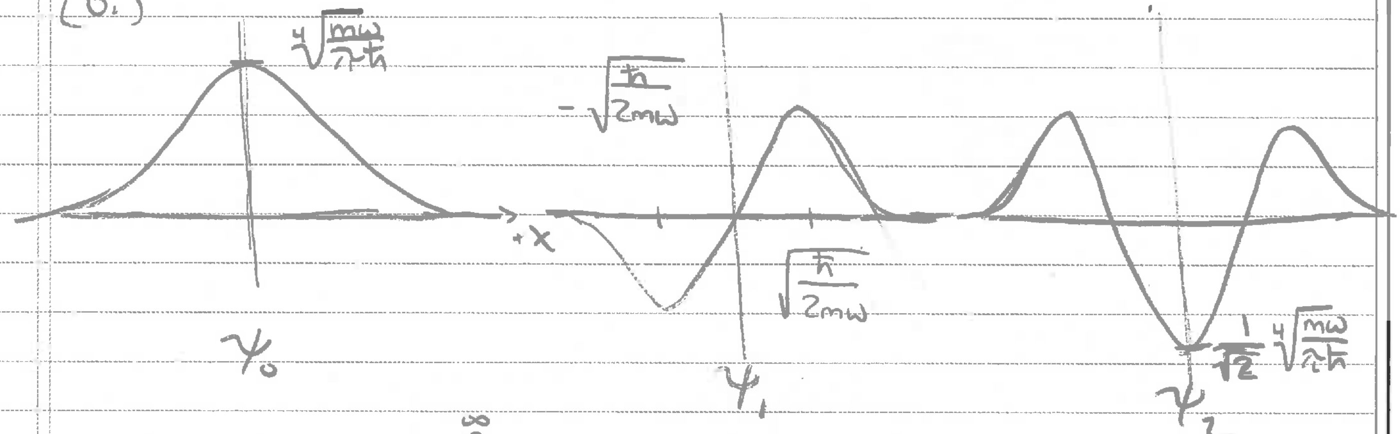
$$\psi_2 = \frac{1}{\sqrt{2}}\hat{a}_+\psi_1 = \frac{1}{\sqrt{2}}\sqrt[4]{\frac{m\omega}{\pi\hbar}}\frac{1}{\sqrt{2\hbar m\omega}}\left(-\hbar\frac{d}{dx} + m\omega x\right)\sqrt{\frac{2m\omega}{\hbar}}\, x\, e^{-\frac{m\omega}{2\hbar}x^2}$$

$$= \frac{1}{\sqrt{2}}\sqrt[4]{\frac{m\omega}{\pi\hbar}}\frac{1}{\hbar}\left(-\hbar\frac{d}{dx}\left[x e^{-\frac{m\omega}{2\hbar}x^2}\right] + m\omega x^2 e^{-\frac{m\omega}{2\hbar}x^2}\right)$$

$$= \frac{1}{\sqrt{2}}\sqrt[4]{\frac{m\omega}{\pi\hbar}}\left(-e^{-\frac{m\omega}{2\hbar}x^2} + 2x^2\left(\frac{m\omega}{2\hbar}\right)e^{-\frac{m\omega}{2\hbar}x^2} + \frac{m\omega}{\hbar}x^2 e^{-\frac{m\omega}{2\hbar}x^2}\right)$$

$$\boxed{\psi_2 = \frac{1}{\sqrt{2}}\sqrt[4]{\frac{m\omega}{\pi\hbar}}\left[\frac{2m\omega}{\hbar}x^2 - 1\right]e^{-\frac{m\omega}{2\hbar}x^2}}$$

(b.)



(c.) $\langle 0|1\rangle = \int_{-\infty}^{\infty}\psi_0^*(x)\psi_1(x)\,dx = 0$

odd integrand
integrated over $(-\infty, \infty)$

$\langle 2|1\rangle = \int_{-\infty}^{\infty}\psi_2^*(x)\psi_1(x)\,dx = 0$

$$\langle 0|2\rangle = \int_{-\infty}^{\infty}\psi_0^*(x)\psi_2(x)\,dx = \frac{1}{\sqrt{2}}\sqrt{\frac{m\omega}{\pi\hbar}}\int_{-\infty}^{\infty}\left(\frac{2m\omega}{\hbar}x^2 - 1\right)e^{-\frac{m\omega}{\hbar}x^2}\,dx$$

$$= \sqrt{\frac{m\omega}{2\pi\hbar}}\left[\underbrace{\int_{-\infty}^{\infty}\frac{2m\omega}{\hbar}x^2 e^{-\frac{m\omega}{\hbar}x^2}\,dx}_{2} - \underbrace{\int_{-\infty}^{\infty}e^{-\frac{m\omega}{\hbar}x^2}\,dx}_{1}\right]$$

2

① $\int_{-\infty}^{\infty} e^{-\frac{m\omega}{\hbar}x^2} dx = \sqrt{\frac{\pi}{\lambda}}$ where $\lambda = \frac{m\omega}{\hbar}$

② $\frac{d}{d\lambda}\int_{-\infty}^{\infty} e^{-\lambda x^2} dx = -\int_{-\infty}^{\infty} x^2 e^{-\lambda x^2} dx$

$\frac{d}{d\lambda}\left(\sqrt{\frac{\pi}{\lambda}}\right) = -\frac{1}{2}\sqrt{\pi}\,\lambda^{-3/2}$

$\rightarrow \int_{-\infty}^{\infty} x^2 e^{-\lambda x^2} dx = \frac{1}{2}\sqrt{\pi}\,\lambda^{-3/2}$

$= \frac{1}{2}\sqrt{\pi}\frac{\hbar}{m\omega}\sqrt{\frac{\hbar}{m\omega}}$

$= \frac{\hbar}{2m\omega}\sqrt{\frac{\pi\hbar}{m\omega}}$

$\langle 0|2\rangle = \sqrt{\frac{m\omega}{2\pi\hbar}}\left(\frac{2m\omega}{\hbar}\frac{1}{2}\sqrt{\pi}\frac{\hbar}{m\omega}\sqrt{\frac{\hbar}{m\omega}} - \sqrt{\pi}\sqrt{\frac{\hbar}{m\omega}}\right)$

$= \sqrt{\frac{m\omega}{2\hbar}}\left(\cancel{\frac{m\omega}{\hbar}}\cancel{\frac{\hbar}{m\omega}}\sqrt{\frac{\hbar}{m\omega}} - \sqrt{\frac{\hbar}{m\omega}}\right) = 0$

2.11 (a) $\langle x\rangle = \langle 0|\hat{x}|0\rangle = \int_{-\infty}^{\infty} \underbrace{\psi_0^*(x)\,x\,\psi_0(x)dx}_{\text{odd integrand}} = 0$

integrated over $(-\infty,\infty)$

$\langle p\rangle = \langle 0|\hat{p}|0\rangle = m\frac{d\langle x\rangle}{dt} = 0$

$\langle x^2\rangle = \langle 0|\hat{x}^2|0\rangle = \int_{-\infty}^{\infty}\psi_0^*(x)\,x^2\,\psi_0(x)dx$

$= \sqrt{\frac{m\omega}{\pi\hbar}}\int_{-\infty}^{\infty} x^2 e^{-\frac{m\omega}{\hbar}x^2} dx = \sqrt{\frac{\cancel{m\omega}}{\pi\hbar}}\,\underline{\sqrt{\frac{\cancel{\pi\hbar}}{m\omega}}\left(\frac{\hbar}{2m\omega}\right)}$

From part (c.) of 2.10

$\langle x^2\rangle = \frac{\hbar}{2m\omega}$

$\langle p^2\rangle = \langle 0|\hat{p}^2|0\rangle = \sqrt{\frac{m\omega}{\pi\hbar}}(-\hbar^2)\int_{-\infty}^{\infty} e^{-\frac{m\omega}{2\hbar}x^2}\left(\frac{d^2}{dx^2}e^{-\frac{m\omega}{2\hbar}x^2}\right)dx$

$\frac{d^2}{dx^2}e^{-\frac{m\omega}{2\hbar}x^2} = \frac{d}{dx}\left(-\frac{m\omega}{\hbar}x\,e^{-\frac{m\omega}{2\hbar}x^2}\right) = \left(\frac{-m\omega}{\hbar} + \frac{m^2\omega^2}{\hbar^2}x^2\right)e^{-\frac{m\omega}{2\hbar}x^2}$

$\langle p^2\rangle = \sqrt{\frac{m\omega}{\pi\hbar}}\int_{-\infty}^{\infty}\hbar m\omega\left(1 - \frac{m\omega}{\hbar}x^2\right)e^{-\frac{m\omega}{\hbar}x^2}dx$

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$$\langle p^2\rangle = \hbar m\omega\sqrt{\frac{m\omega}{\pi\hbar}}\left[\sqrt{\pi}\underbrace{\left(\sqrt{\frac{\hbar}{m\omega}} - \frac{1}{2}\frac{m\omega}{\hbar}\frac{\hbar}{m\omega}\sqrt{\frac{\hbar}{m\omega}}\right)}_{1-1/2}\right]$$

$$\boxed{\langle p^2\rangle = \hbar m\omega/2}$$

$$\xi = \sqrt{\frac{m\omega}{\hbar}}\,x \longrightarrow \psi_1(\xi) = \alpha\sqrt{2}\,\xi\, e^{-\xi^2/2}$$

$$\alpha = \sqrt[4]{\frac{m\omega}{\pi\hbar}}$$

$$d\xi = \sqrt{\frac{m\omega}{\hbar}}\,dx$$

$$\langle x\rangle = \langle\psi_1|\hat{x}|\psi_1\rangle = \int_{-\infty}^{\infty}\alpha^2\, 2\,\xi^3\left(\sqrt{\frac{\hbar}{m\omega}}\right)^2 e^{-\xi^2}\,d\xi$$

$$= 2\alpha^2\frac{\hbar}{m\omega}\int_{-\infty}^{\infty}\underbrace{\xi^3 e^{-\xi^2}\,d\xi}_{\text{odd integral}} = 0$$

odd integral integrated over $(-\infty,\infty)$

$$\langle p\rangle = \langle\psi_1|\hat{p}|\psi_1\rangle = m\frac{d\langle x\rangle}{dt} = 0$$

$$\langle x^2\rangle = \langle\psi_1|\hat{x}^2|\psi_1\rangle = \int_{-\infty}^{\infty}2\alpha^2\left(\frac{\hbar}{m\omega}\right)^{3/2}\xi^4 e^{-\xi^2}\,d\xi$$

$$\frac{d}{d\lambda}\int_{-\infty}^{\infty}x^2 e^{-\lambda x^2}\,dx = -\int_{-\infty}^{\infty}x^4 e^{-\lambda x^2}\,dx \quad\longrightarrow\quad \int_{-\infty}^{\infty}x^4 e^{-\lambda x^2}\,dx = \frac{3}{4}\frac{1}{\lambda^2}\sqrt{\frac{\pi}{\lambda}}$$

$$\frac{d}{d\lambda}\left(\frac{1}{2}\sqrt{\pi}\,\lambda^{-3/2}\right) = -\frac{3}{4}\sqrt{\pi}\,\lambda^{-5/2}$$

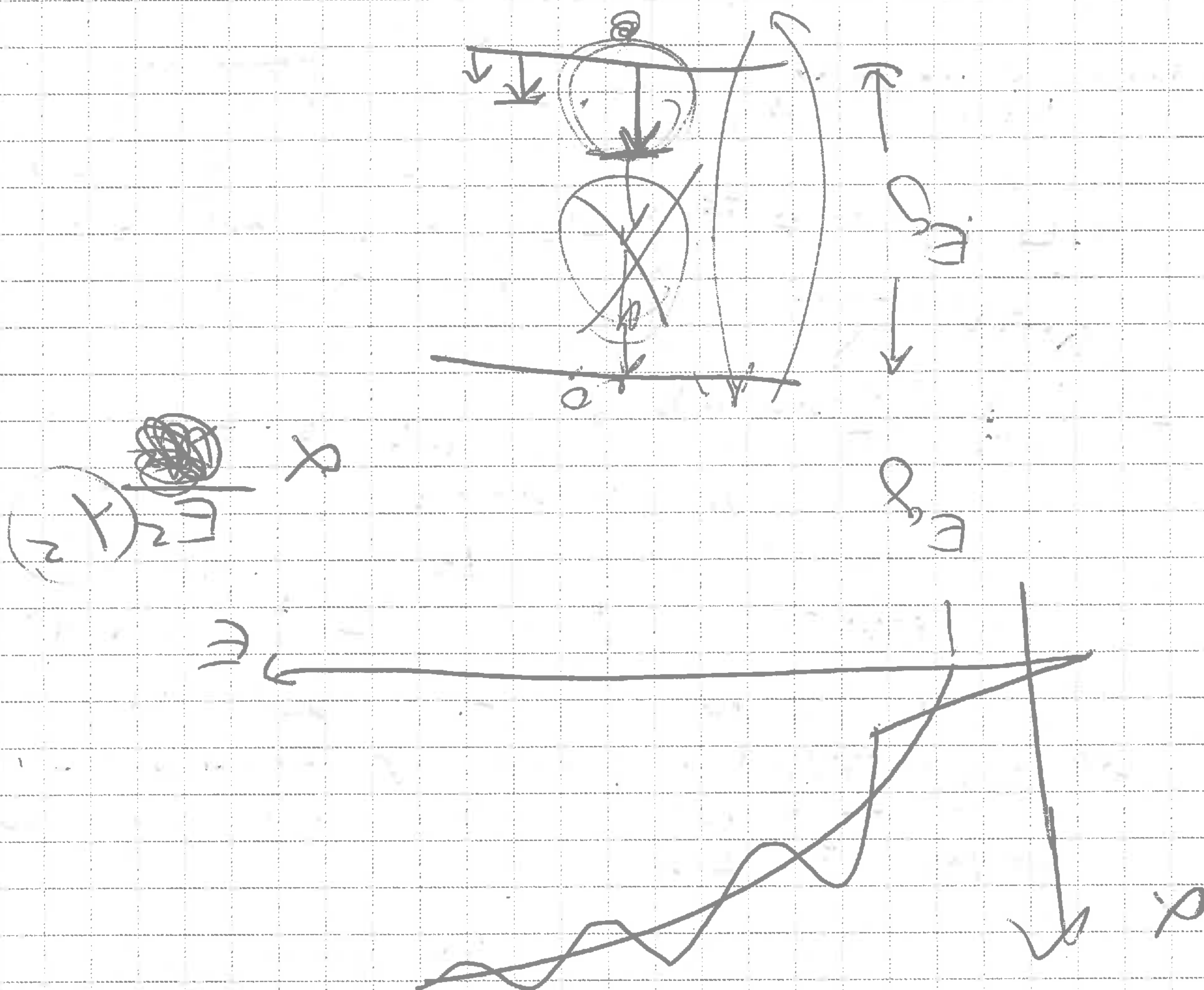
$$\langle x^2\rangle = 2\alpha^2\left(\frac{\hbar}{m\omega}\right)^{3/2}\frac{3}{4}\sqrt{\pi} = \frac{3}{2}\sqrt{\pi}\sqrt{\frac{m\omega}{\pi\hbar}}\left(\frac{\hbar}{m\omega}\right)\sqrt{\frac{\hbar}{m\omega}} = \frac{3\hbar}{2m\omega}$$

$$\langle p^2\rangle = \langle\psi_1|\hat{p}^2|\psi_1\rangle = 2\alpha^2(-\hbar^2)\int_{-\infty}^{\infty}\xi e^{-\xi^2/2}\frac{d^2}{d\xi^2}\left(\xi e^{-\xi^2/2}\right)d\xi\left(\sqrt{\frac{m\omega}{\hbar}}\right)$$

$$\frac{d^2}{d\xi^2}\left(\xi e^{-\xi^2/2}\right) = \frac{d}{d\xi}\left(e^{-\xi^2/2} - \cancel{2}\xi^2 e^{-\xi^2/2}\right) = -\xi e^{-\xi^2/2} - 2\xi e^{-\xi^2} + \xi^3 e^{-\xi^2/2}$$

$$\langle p^2\rangle = -2\hbar^2\alpha^2\sqrt{\frac{m\omega}{\hbar}}\int_{-\infty}^{\infty}e^{-\xi^2}\left(-3\xi^2 + \xi^4\right)d\xi$$

$$= 2\hbar^2\alpha^2\sqrt{\frac{m\omega}{\hbar}}\left[3\left(\frac{1}{2}\sqrt{\pi}\right) - \frac{3}{4}\sqrt{\pi}\right] = 2\hbar^2\sqrt{\frac{m\omega}{\pi\hbar}}\sqrt{\frac{m\omega\pi}{\hbar}}\left[\frac{3}{2} - \frac{3}{4}\right]$$



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$$\langle p^2\rangle = 2\frac{3}{4}\hbar^2\frac{m\omega}{\hbar} = \boxed{\frac{3}{2}m\hbar\omega}$$

(b.)

ψ_0: $\sigma_x = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = \sqrt{\frac{\hbar}{2m\omega}}$

$\sigma_p = \sqrt{\langle p^2\rangle - \langle p\rangle^2} = \sqrt{\frac{m\hbar\omega}{2}}$

$\sigma_x\sigma_p = \sqrt{\frac{\hbar}{2m\omega}}\sqrt{\frac{m\hbar\omega}{2}} = \hbar/2$ (Directly @ uncertainty limit)

ψ_1: $\sigma_x = \sqrt{\frac{3\hbar}{2m\omega}}$

$\sigma_p = \sqrt{\frac{3}{2}m\hbar\omega}$

$\sigma_x\sigma_p = \frac{3}{2}\hbar$ which is $> \hbar/2$

(c.)

ψ_0: $\langle T\rangle = \frac{\langle p^2\rangle}{2m} = \frac{1}{2m}\frac{\hbar m\omega}{2} = \frac{\hbar\omega}{4}$ $\quad E_0 = \frac{\hbar\omega}{2} = \langle T\rangle + \langle V\rangle$ ✓

$\langle V\rangle = \frac{1}{2}m\omega^2\langle x^2\rangle = \frac{1}{2}m\omega^2\frac{\hbar}{2m\omega} = \frac{\hbar\omega}{4}$

ψ_1: $\langle T\rangle = \frac{1}{2m}\frac{3}{2}m\hbar\omega = \frac{3\hbar\omega}{4}$ $\quad E_1 = \frac{3\hbar\omega}{2} = \langle T\rangle + \langle V\rangle$ ✓

$\langle V\rangle = \frac{1}{2}m\omega^2\left(\frac{3\hbar}{2m\omega}\right) = \frac{3\hbar\omega}{4}$

2.12

$$\langle x\rangle = \langle n|\sqrt{\frac{\hbar}{2m\omega}}(\hat{a}_+ + \hat{a}_-)|n\rangle = \sqrt{\frac{\hbar}{2m\omega}}\left[\langle n|\hat{a}_+|n\rangle + \langle n|\hat{a}_-|n\rangle\right]$$

$$= \sqrt{\frac{\hbar}{2m\omega}}\left[\underbrace{\langle n|\sqrt{n+1}|n+1\rangle}_{0} + \underbrace{\langle n|\sqrt{n}|n-1\rangle}_{0}\right] = 0$$

$$\langle p\rangle = \langle n|i\sqrt{\frac{\hbar m\omega}{2}}(\hat{a}_+ - \hat{a}_-)|n\rangle = 0 \quad \text{(same as above)}$$

$$\langle x^2\rangle = \langle n|\hat{x}^2|n\rangle \; : \; \hat{x}^2 = \frac{\hbar}{2m\omega}\left(\hat{a}_+^2 + \hat{a}_+\hat{a}_- + \hat{a}_-\hat{a}_+ + \hat{a}_-^2\right)$$

$$= \frac{\hbar}{2m\omega}\left[\underbrace{\langle n|\hat{a}_+^2|n\rangle}_{0} + \underbrace{\langle n|\hat{a}_+\hat{a}_-|n\rangle}_{n\langle n|n\rangle} + \underbrace{\langle n|\hat{a}_-\hat{a}_+|n\rangle}_{(n+1)\langle n|n\rangle} + \underbrace{\langle n|\hat{a}_-^2|n\rangle}_{0}\right]$$

$$\langle x^2\rangle = \frac{\hbar}{2m\omega}(n+n+1) = \boxed{\frac{\hbar}{m\omega}\left(n+\frac{1}{2}\right)}$$

$$\langle p^2\rangle = \langle n|\hat{p}^2|n\rangle$$

$$\hat{p}^2 = -\frac{\hbar m\omega}{2}\left(\hat{a}_+^2 - \hat{a}_+\hat{a}_- - \hat{a}_-\hat{a}_+ + \hat{a}_-^2\right)$$

$$\langle p^2\rangle = \frac{-\hbar m\omega}{2}\Big[\underbrace{\langle n|\hat{a}_+^2|n\rangle}_{0} - \underbrace{\langle n|\hat{a}_+\hat{a}_-|n\rangle}_{n\langle n|n\rangle} - \underbrace{\langle n|\hat{a}_-\hat{a}_+|n\rangle}_{(n+1)\langle n|n\rangle} + \underbrace{\langle n|\hat{a}_-^2|n\rangle}_{0}\Big]$$

$$\boxed{\langle p^2\rangle = \hbar m\omega\left(n+\frac{1}{2}\right)}$$

$$\langle T\rangle = \frac{\langle p^2\rangle}{2m} = \boxed{\frac{\hbar\omega}{2}\left(n+\frac{1}{2}\right)}$$

$$\sigma_x = \sqrt{\frac{\hbar}{m\omega}\left(n+\frac{1}{2}\right)} \qquad \sigma_p = \sqrt{\hbar m\omega\left(n+\frac{1}{2}\right)}$$

$$\sigma_x\sigma_p = \hbar\left(n+\frac{1}{2}\right)$$

Min. @ $n=0$, where $\sigma_x\sigma_p = \hbar/2$ (right @ uncertainty limit)

For all other values of n (i.e., $n>0$), $\sigma_x\sigma_p > \hbar/2$.

2.17

(a.) $H_n(\xi) = (-1)^n e^{\xi^2}\left(\frac{d}{d\xi}\right)^n e^{-\xi^2}$

$$H_3 = (-1)e^{\xi^2}\frac{d^3}{d\xi^3}e^{-\xi^2} = (-1)e^{\xi^2}\underline{(-8\xi^3+12\xi)e^{-\xi^2}}$$

$$\frac{d^2}{d\xi^2}\left(-2\xi e^{-\xi^2}\right) = \frac{d}{d\xi}\left([4\xi^2-2]e^{-\xi^2}\right) = (8\xi-8\xi^3+4\xi)e^{-\xi^2}$$

$$\boxed{H_3 = 8\xi^3 - 12\xi}$$

$$H_4 = +e^{\xi^2}\frac{d^4}{d\xi^4}e^{-\xi^2}$$

$$\frac{d^4}{d\xi^4}e^{-\xi^2} = \frac{d}{d\xi}\left[(12\xi-8\xi^3)e^{-\xi^2}\right] = \left[(12-24\xi^2)+(-24\xi^2+16\xi^4)\right]e^{-\xi^2}$$

$$= (16\xi^4 - 48\xi^2 + 12)e^{\xi^2}$$

$$\boxed{H_4 = 16\xi^4 - 48\xi^2 + 12}$$

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(b.) $H_{n+1}(\xi) = 2\xi H_n(\xi) - 2n H_{n-1}(\xi)$

$H_5(\xi) = 2\xi H_4(\xi) - 2(4) H_3(\xi)$ $\quad n=4$

$H_5(\xi) = 32\xi^5 - 96\xi^3 + 24\xi - 8(8\xi^3 - 12\xi)$

$\boxed{H_5(\xi) = 32\xi^5 - 160\xi^3 + 120\xi}$

$H_6(\xi) = 2\xi(H_5) - 10(H_4)$ $\quad n=5$

$= 64\xi^6 - 320\xi^4 + 240\xi^2 - 160\xi^4 + 480\xi^2 - 120$

$\boxed{H_6(\xi) = 64\xi^6 - 480\xi^4 + 720\xi^2 - 120}$

(c.) $\frac{dH_n}{d\xi} = 2n H_{n-1}(\xi)$

n=5: $\frac{dH_5}{d\xi} = 10 H_4(\xi) = 160\xi^4 - 480\xi^2 + 120$ ✓

$\frac{dH_5}{d\xi} = 5(32\xi^4) - 3(160\xi^2) + 120 = 160\xi^4 - 480\xi^2 + 120$ ✓

n=6 $\frac{dH_6}{d\xi} = 12 H_5(\xi) = 12\left[32\xi^5 - 160\xi^3 + 120\xi\right]$

$12 H_5(\xi) = 384\xi^5 - 1920\xi^3 + 1440\xi$

$\frac{dH_6}{d\xi} = 6(64\xi^5) - 4(480\xi^3) + 2(720\xi)$

$\frac{dH_6}{d\xi} = 384\xi^5 - 1920\xi^3 + 1440\xi$ ✓

(d.) n^{th} z-derivative of $\exp(-z^2 + 2z\xi)$ @ $z=0$

H_0 (n=0)

no derivative of $e^{-z^2+2z\xi}$. Evaluating @ $z=0$ gives $\boxed{H_0(\xi) = 1.}$

H_1 (n=1): $\frac{d}{dz}\left(e^{-z^2+2z\xi}\right) = (-2z + 2\xi)e^{-z^2+2z\xi}\Big|_{z=0} = 2\xi$

$\boxed{H_1(\xi) = 2\xi}$

H_2 (n=2); $\frac{d}{dz}\left[(-2z+2\xi)e^{-z^2+2z\xi}\right] = \left[(-2z+2\xi)^2 - 2\right]e^{-z^2+2z\xi}\Big|_{z=0}$

$$\boxed{H_2(\xi) = 4\xi^2 - 2}$$

4.13

(a.) $V(r) = \frac{-e^2}{r} + \frac{A}{r^{n}} = -e^2 r^{-1} + A r^{-n}$

$$\left.\frac{\partial V}{\partial r}\right|_{r_0} = 0 = e^2 r^{-2} - nAr^{-(n+1)}\Big|_{r=r_0}$$

$$\boxed{r_0 = \left(\frac{An}{e^2}\right)^{\frac{1}{n-1}}}$$

Later, we will need to put everything in terms of n, so let's isolate A:

$$A = \frac{e^2}{n} r_0^{n-1}$$

(b.)

V_{min} occurs @ r_0. Or put another way, in our harmonic oscillator model, we Taylor expanded $V(r)$:

$$V(r) = V(r_0) + V'(r_0)(r-r_0) + \tfrac{1}{2}V''(r_0)(r-r_0)^2 + \ldots$$

At a min., $V'(r_0) = 0$ leaving only a constant, $V(r_0)$, and a spring term: $\frac{1}{2}V''(r_0)(r-r_0)^2 = \frac{1}{2}C(r-r_0)^2$

$$\boxed{V_{min} = V(r_0) = \frac{-e^2}{r_0} + Ar_0^{-n} = -e^2\left(\frac{An}{e^2}\right)^{\frac{-1}{n-1}} + A\left(\frac{An}{e^2}\right)^{\frac{-n}{n-1}}}$$

(c.)

$$\left.\frac{\partial^2 V}{\partial r^2}\right|_{r=r_0} = -2e^2 r_0^{-3} + n(n+1)Ar_0^{-(n+2)}$$

$$\boxed{\left.\frac{\partial^2 V}{\partial r^2}\right|_{r=r_0} = -2e^2\left(\frac{An}{e^2}\right)^{\frac{-3}{n-1}} + n(n+1)A\left(\frac{An}{e^2}\right)^{-\left(\frac{n+2}{n-1}\right)} = C}$$

(d.) $\mu = \left(\frac{1}{m_1} + \frac{1}{m_2}\right)^{-1} = \frac{m_1 m_2}{m_1 + m_2}$

$Na^{23} = 23(1.66\times10^{-24}\,g)$

$Cl^{35} = 35(1.66\times10^{-24}\,g)$

$$\mu = \left(\frac{23\times35}{23+35}\right)1.66\times10^{-24}\,g = 4.28\times10^{-23}\,g$$

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Energy spacing $= \hbar\omega_0 = \hbar\sqrt{C/\mu}$

$\mu^{-1/2} = 1.53\times10^{11}\ g^{-1/2}$ $\qquad \hbar = 1.055\times10^{-27}\ cm^2\text{-}g/s$

$$\hbar\omega_0 = 1.055\times10^{-27}\ \frac{cm^2\text{-}g}{s}\times1.53\times10^{11}\ g^{-1/2}\left[-2e r_0^{-3} + n(n+1)A r_0^{-(n+2)}\right]^{1/2}$$

$$\boxed{\hbar\omega_0 = 1.61\times10^{-16}\ \frac{cm^2\text{-}g^{1/2}}{s}\left[-2e^2 r_0^{-3} + n(n+1)A r_0^{-(n+2)}\right]^{1/2}}$$

$$\boxed{\text{Zero-pt. energy} = \frac{\hbar\omega_0}{2} = 8.06\times10^{-17}\ \frac{cm^2\text{-}g^{1/2}}{s}\left[-2e^2 r_0^{-3} + n(n+1)A r_0^{-(n+2)}\right]^{1/2}}$$

(e.) $r_0 = 2.51\ \text{Å} = 2.51\times10^{-8}\ cm$

$\nu_0 = 1.14\times10^{13}\ Hz \Rightarrow \omega_0 = 2\pi\nu_0 = 7.16\times10^{13}\ rad/sec.$

Let's first determine $C\ (=\mu\omega_0^2)$

$$C = (4.28\times10^{-23}\ g)(7.16\times10^{13}\ rad/sec)^2 = 1.18\times10^{5}\ dynes/cm$$

Next, using $\left.\frac{\partial^2 V}{\partial r^2}\right|_{r=r_0} = C$, let's isolate for A and n.

$$\left.\frac{\partial^2 V}{\partial r^2}\right|_{r=r_0} = C = -2e^2 r_0^{-3} + n(n+1)A r_0^{-(n+2)}$$

$$C + \frac{2e^2}{r_0^3} = n(n+1)A r_0^{-(n+2)}$$

We now have a constant on the LHS of the eqn and two variables on the RHS. Using our result from part (c.), namely $A = \frac{e^2}{n} r_0^{n-1}$, we can greatly simplify our eqn and isolate for n.

$$C + \frac{2e^2}{r_0^3} = n(n+1)\frac{e^2}{n} r_0^{n+1} r_0^{-n-2}$$

$$C + \frac{2e^2}{r_0^3} = (n+1)e^2 r_0^{-3}$$

$$\frac{C r_0^3}{e^2} + 2 = n+1 \Rightarrow \boxed{n = \frac{C r_0^3}{e^2} + 1}$$

$$n = \frac{1.18\times10^5\ dynes/cm}{(4.80\times10^{-10}\ statC)^2}(2.51\times10^{-8}\ cm)^3 + 1 = 9.11$$

$$A = \frac{e^2}{n} r_0^{n-1} = \frac{(4.80\times10^{-10}\ statC)^2}{9.11}(2.51\times10^{-8}\ cm)^{8.11} = \boxed{5.47\times10^{-82}}$$

(f.)

$$V_{min} = V(r_0) = \frac{-e^2}{r_0} + A r_0^{-n}$$

$$= -\frac{(4.80\times10^{-10}\ \text{statC})^2}{2.51\times10^{-8}\ \text{cm}} + 5.47\times10^{-82}\,(2.51\times10^{-8}\ \text{cm})^{-9.11}$$

$$\boxed{V_{min} = -8.17\times10^{-12}\ \text{erg} \quad \text{or} \quad -5.1\ \text{eV}}$$

$$\hbar\omega_0 = 1.61\times10^{-16}\left[-2e^2/r_0^3 + n(n+1)\,A/r_0^{n+2}\right]^{1/2}$$

$$\boxed{\hbar\omega_0 = 5.33\times10^{-14}\ \text{erg} = 0.0333\ \text{eV}}$$

$$\text{Dissociation energy} = -\left[\frac{\hbar\omega_0}{2} + V_{min}\right]$$

$$= 5.1\ \text{eV}$$

$$NaCl + 5.1\ \text{eV} \longrightarrow Na^{\oplus} + Cl^{\ominus}$$

(g.)

$$Na + 5.1\ \text{eV} \longrightarrow Na^{\oplus} + e^{\ominus}$$

$$Cl + e^{\ominus} \longrightarrow Cl^{\ominus} + 3.7\ \text{eV}$$

$$Na + Cl + \underbrace{1.4\ \text{eV}}_{\text{experimental}} \longrightarrow Na^{\oplus} + Cl^{\ominus}$$

In part (f.), we found that:

$$NaCl + \underbrace{5.1\ \text{eV}}_{\text{theoretical}} \longrightarrow Na^{\oplus} + Cl^{\ominus}$$

We want to find:

$$NaCl + \Delta \longrightarrow Na + Cl$$

$$NaCl + 5.1\ \text{eV} \longrightarrow Na^{\oplus} + Cl^{\ominus}$$

$$Na^{\oplus} + Cl^{\ominus} - 1.4\ \text{eV} \longrightarrow Na + Cl$$

$$NaCl + 3.7\ \text{eV} \longrightarrow Na + Cl$$

We need to input 3.7 eV of energy into NaCl to dissociate it into Na and Cl atoms. This value is only 14% (~1 part in 7) off the empirical value of 4.3 eV.