PHYS 5130, Homework Set 2, Part II, due at 5 pm on Thursday, Mar. 3rd.

Group work (up to three members) is allowed; only turn in one version of the homework with all names included.

1. In class, we discussed how a static magnetic dipole moment, $\vec{\mu}_m(t) = \int_V \vec{r} \times \vec{J}_e(\vec{r},t)\,dV$, could be non-zero for a single quantum state. Here:

$$\vec{J}_e(\vec{r},t) = -\frac{ie\hbar}{2m}\left(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi\right),$$

is the current density for an electron.
Find $\vec{\mu}_m(t)$ for $\Psi = \frac{1}{\sqrt{2}}\left(\Psi_{210} + \Psi_{211}\right)$ in hydrogen. This scenario represents a time-varying, or precessing, dipole moment.

Hint 1: $\Psi_{nlm} \to \Psi_{210} = R_{21}Y_1^0\Phi_1(t) = \frac{1}{4a^{5/2}}\sqrt{\frac{1}{2\pi}}re^{-r/2a}\cos\theta e^{-i\omega_1 t}$. Similarly, $\Psi_{211} = R_{21}Y_1^1\Phi_2(t) = \frac{-1}{8a^{5/2}}\sqrt{\frac{1}{\pi}}re^{-r/2a}\sin\theta e^{i\phi}e^{-i\omega_2 t}$. Here, a is the Bohr radius, and ω_i is equal to $E_i/\hbar$, where E_i is the energy eigenstate.

Hint 2: Keeping separation of variables may be helpful. For example, $\Psi = Af(r)\,g(\theta)\,h(\phi)\,j(t)$.

Hint 3: What is $\mathbf{r}\times\nabla$, where $\mathbf{r} = r\hat{r}$ and ∇ is the gradient in spherical coordinates?

Hint 4: The solution should have a precession component, oscillating at a frequency of $\omega_{21} = \omega_2 - \omega_1$, in the $x-y$ plane and a static component in the z plane.

2. Is spin important for this problem? If we included χ (the two-element spin column matrix) into Ψ, what would change?